

14 Dynamics and the Problem of Visual Event Recognition

Geoffrey P. Bingham

EDITOR'S INTRODUCTION

How is visual perception possible? In particular, how is it that what one typically sees is a relatively simple world of objects and behaviors, when what reaches the eyes is a fantastically rich, seemingly chaotic play of stimulation? How does one's visual system manage to reach behind the superficial confusion to the stability and order that are responsible for it?

In this chapter, Geoff Bingham confronts one version of this problem, that of recognition of events. We constantly perceive what is going on around us as meaningful events of certain kinds: a person walking, a ball bouncing, water flowing. Psychologists have established experimentally that people are very good at recognizing the nature of an event from the visual motions the event produces; thus it is easy to see that a flow of light patches against a dark background is produced by a ball bouncing. The event itself is determined by a characteristic dynamics; thus the laws of classical mechanics determine the motion of a bouncing ball. The problem of event recognition is to recover the dynamics of the event from the visual motions, i.e., the kinematics.

One problem in event recognition is that researchers have believed the motions to be ambiguous; the same surface motions might have been produced by many kinds of dynamics. A standard approach to the difficulty of narrowing down the search has been to use the assumption that only rigid objects are involved. Yet, as Bingham points out, events involving rigid objects are just one kind among many that we can distinguish; hence the rigidity assumption is a dead end. Acknowledging this, however, seems to render the problem insoluble. There must be some further kind of structure or information that we rely on in recognizing events.

If we think of the sensory periphery as a kind of boundary between inner and outer, then cognitive scientists can proceed in at least two ways. Traditionally, they focus on what is inside this boundary, on the states and processes that supposedly enable a cognizant agent to piece together an interpretation of the world based on impoverished sensory data. On this approach, the further information that is needed to solve the problem of event recognition must take the form of internally represented background knowledge which is brought to bear in complex computational operations.

An alternative approach is to focus on what is outside the boundary. Perhaps there is already present in the sensory stimulation information enabling the system to identify the nature of the event. If this is right, the task of the visual system would just be to pick up on that information; the need for internal representations and computations would be minimized. From this perspective, then, an essential preliminary to investigating the internal cognitive mechanisms involved in visual perception is to develop, as Bingham puts it, "a job description for the sensory apparatus."

In this chapter, Bingham takes this second approach, and argues that the dynamics of the event is not, in fact, as hopelessly underspecified by the kinematics as might be supposed. Natural events are constrained by natural law, and hence the motions that result reflect certain universally valid circumstances such as the constancy of gravitational force and the unidirectionality of time. Further, if one adopts a suitably global perspective (i.e., one that accords with the time scale of the complete event itself), then there exist symmetries in the temporally extended pattern of sensory stimulation that further constrain the nature of the event that could have produced it.

Bingham substantiates these points with extended analysis of a particular example, that of a ball rolling back and forth inside a U-shaped container. If one visually tracks the movement of individual points on this ball, the result is a myriad of short, disjointed trajectories. The problem is to show that this information, together with further ecologically valid assumptions, uniquely constrains the nature of the event responsible (a ball rolling); or, to put the point another way, that under normal ecological conditions, the mapping from the dynamics of an event to the kinematics of the optic array is unique and reversible. The cognitive task of event identification appears far less daunting if this bold claim can be substantiated.

14.1 INTRODUCTION

People are able to recognize an indefinite variety of events visually. Motions in events have been shown to provide the information. The question is, What optical information do people use to recognize events, that is, how do motions get into optics? For instance, consider the visual perception of a ball rolling over a surface. This event can be readily recognized even when it appears in a video display in which only small bright irregular patches on the ball are visible in darkness. In analyzing the perception of this event, we must be careful to distinguish between the event and the optics. *In the event itself*, each patch follows a continuous trajectory along a path of particular shape and with a velocity that varies in a particular way along the path. Each patch follows a path of somewhat different shape with a somewhat different velocity pattern and each of these patterns may be shifted somewhat spatially relative to the others along the surface. How are all of these distinct patch trajectories combined to yield the perception of a unitary coherent event? The problem is more difficult than this, however. In the display, the patches blink on and off. They appear and disappear as they roll up over the top of the ball and then around behind the ball. *In the optics*, a patch follows a

discontinuous piece of a trajectory. Each trajectory piece has a somewhat different path shape and velocity pattern and each piece is spatially shifted with respect to all of the other pieces across the display. Most important, pieces sampled successively from the trajectory of a single patch in the event itself cannot readily be identified as such. Each sampled piece from a given event trajectory is separated by a relatively large distance from the preceding and following pieces. Neighboring trajectories are arbitrarily close and may be easily confused. The optics consists, therefore, of a very large collection of qualitatively distinct and spatially disparate trajectory pieces. Nevertheless, this extremely complex mess is perceived simply as a single rolling ball. How is this possible? Clearly, the collection of trajectory pieces must be structured and the perceptual system must detect and use that structure.

The difficulty is that events are inherently time extended so that the structure used to identify events must also be time extended. Historically, the trend in analysis of optical structure has been away from structure that is strongly local in space and time toward more global structures. This trend has been largely motivated by the intractability of the problems formulated on the basis of very local structure. The optical array is used to describe the pattern in light projected from all directions to a point of observation. Optical flow is the changing pattern produced when the point of observation moves or when surfaces in the environment around a point of observation move. The optical array was introduced by Gibson (1961) to emphasize spatially extended structure surrounding an observer and to provide a means of capturing optical flow. With the introduction of optical flow, the relevant structure became extended in time beyond instantaneous snapshots. However, the extension in time has only progressed in the majority of extant analyses to a sequence of two or three images obtained over a few milliseconds and yielding an extremely brief sample of optical flow over distances within an infinitesimal neighborhood of a point in the flow field. Because of the strongly local character of these measurements, the results of the analyses have not been stable in the face of perturbations representing noise engendered by the sensory apparatus. An assumption that an event consists of strictly rigid motions has been used in an attempt to make analysis less local. Rigidity of motion means that distances between points in three-dimensional space are preserved so that the motions of a given point constrain those of neighboring points. However, recent investigations have shown that only truly global analysis will resolve these difficulties (Bertero, Poggio and Torre, 1988; Eagleson, 1987; Hildreth and Grzywacz, 1986; Hildreth and Koch, 1987; Jacobson and Wechsler, 1987; Nagel, 1988; Ullman, 1984; Verri and Poggio, 1987, 1989).

A global analysis is advocated in this chapter for a different but related reason. To assume rigid motion is to beg the question of event recognition. Rigid motion is but one of many types of motion that can occur in a wide variety of distinct types of recognizable events. Such motions include, for instance, elastic, plastic, liquid, or ethereal motions, among others. Truly time-extended information is required to enable recognition of these types of

events. For instance, imagine trying to distinguish among the following events in which irregular tickets of white paper have been used as patches appearing in otherwise dark displays: patches on the facial skin of a talking person, patches on the surface of vibrating jello, patches on the surface of a trampoline during gymnastic exercises, patches on the surface of water being stirred or splashed by a projectile, patches on a handful of coins being slid across the bottom of a wooden box, patches being blown across a surface like leaves blown across a lawn in autumn, and patches on a collection of Ping-Pong balls dropped on a tile floor. All of these events involve nonrigid motions. All might be distinguished with sufficiently time-extended samples. Each involves different physical constraints on the motions of the patches. Each, accordingly, involves a distinct type of motion. The challenge is to characterize the motions in a way that enables us to begin to formulate the event recognition problem.

For the purpose of providing an initial outline of the problem of event recognition, I will characterize events in terms of *trajectory forms in phase space* (in which velocities are plotted against position).¹ Characterized in this way, *events appear as spatiotemporal objects* that can be mapped via perspective projections into an optical phase space of lower dimension. Events then can be distinguished on the basis of qualitative properties.

I begin by reviewing the evidence on event recognition via forms of motion. Next, I consider how mere motions can provide information about the substantial types and properties of events. To anticipate briefly, formulation in terms of the qualitative properties of trajectories allows one to use qualitative dynamics to capture, in a single qualitative characterization, both the substantial properties of events (in terms of dynamics) and the information about them (in terms of kinematics). Geometrically, dynamics corresponds to vector fields in phase space while the kinematics are trajectories in phase space. The forms of dynamic vector fields are identical to the forms of the kinematic trajectories that lie tangent to them and are determined by them. Under this abstract and qualitative way of construing dynamics, kinematics (i.e., motions) and dynamics (i.e., physical properties) are commensurate and kinematics can specify dynamics. Along the way, I describe the relation between the kinematic specification of dynamics and the notion of direct perception.

Next, an example, namely, a ball rolling on a curved surface, is used to illustrate the problems engendered by the projection of event phase-space trajectories into an optical phase space of lower dimension. The question is, What qualitative properties are preserved in the projection to optical flows? Finally, the ultimate difficulty, the degrees-of-freedom problem, is discussed together with methods of qualitative dynamics that can be used to solve it. The degrees of freedom are the separate items that must be measured (or apprehended) and evaluated. The difficulty, as I have already indicated, is that occlusion yields disconnected pieces of trajectories in the optics. When counted, these pieces amount to an excessively large number of potential

degrees of freedom. The problem of perceptual organization, as formulated by the Gestalt psychologists, must here be confronted. To anticipate, I suggest that a solution can be found in the qualitative properties of event trajectories. Symmetries² apparent in the form and layout of trajectory pieces can be used to collapse the pieces together into temporally continuous and spatially coherent forms, reducing the degrees of freedom and revealing information that could be used for recognition.

14.2 THE EVIDENCE FOR EVENT RECOGNITION

Evidence has been amassed over the last 30 to 40 years demonstrating irrefutably that people are able to recognize specific types of events and specific properties of events via detection of particular forms of motion. The majority of the extant research in visual event perception has been focused on scaling problems, that is, the way that magnitudes associated with particular event properties are apprehended. This research has included investigations on the perception of the sizes and distances of objects in free fall (Johansson and Jansson, 1967; Muchisky and Bingham, 1992; Watson, Banks, von Hofsten, et al., 1993); perception of the lengths of swinging pendulums (Pittenger, 1985, 1990); perception of amounts of lifted weight (Runeson and Frykholm, 1981, 1983; Bingham, 1985, 1987b); perception of relative amounts of mass in collisions (Proffitt and Gilden, 1989; Runeson, 1977; Runeson and Vedeler, 1993; Todd and Warren, 1982); perception of the age of growing heads (Mark, Todd, and Shaw, 1981; Pittenger and Shaw, 1975; Shaw, Mark, Jenkins, et al., 1982; Shaw and Pittenger, 1977, 1978; Todd, Mark, Shaw, et al., 1980); perception of the elasticity of bouncing balls (Warren, Kim, and Husney, 1987); and perception of the time of contact of projectiles (Lee, Young, Reddish, et al., 1983; Todd, 1981). All of these scaling studies have implicitly involved the problem of recognition because any property or dimension to be scaled must first be recognized. For instance, to judge pendulum length via the period requires that an observer recognize the freely swinging pendulum event as well as the event property, pendulum length. Successful performance in all of the cited scaling studies has implied that observers have been able to recognize the event properties whose scale values they judged. To this may be added evidence from investigations explicitly on recognition.

The inaugural studies on visual event recognition include those of Duncker, Michotte, Wallach, and Johansson. Duncker (1929/1950) demonstrated the recognition of a rigid rolling wheel via the relative motions of points on the hub and the rim. Michotte (1963) studied the recognition of launching vs. triggering events as the timing along trajectories was varied. Wallach and O'Connell (1953) investigated the recognition of wire frame objects via the so-called kinetic depth effect. Finally, Johansson (1950), in giving event perception research its name, placed it in the context of established problems in perceptual research, namely those of perceptual organization and constancy. Manipulating the motions of points or elements in a two-dimensional display,

Johansson sought properties of relative motions that would result in the perception of a single coherent moving three-dimensional object. In addition, Johansson distinguished between displays that yielded perception of rigid vs. nonrigid objects and inquired as to the conditions yielding the shape constancy of rigid objects (Johansson, 1950, 1964, 1973, 1985). This led to an entire area of research on object recognition called "structure-from-motion" in which the assumption of "rigid motion" has been used in theorems proving that three-dimensional object structure can be derived from sampled optical transformations (Hildreth, 1984; Hildreth and Hollerbach, 1987; Hildreth and Koch, 1987; Longuet-Higgins and Prazdny, 1980; Marr, 1982; Prazdny, 1980; Ullman, 1979).

"Structure-from-motion" research owes as much to Gibson's studies on the visual control of locomotion and flight (e.g., Gibson, 1955, 1958, 1961, 1966; Gibson, Gibson, Smith, et al., 1959) as to Johansson's studies on event perception. The rigid/nonrigid distinction has been used to investigate perspective transformations that occur as a point of observation is moved through the environment. The assumption that the environment should be entirely rigid (and therefore static) yields a reasonable first approximation to optical flows encountered during locomotion (Nayakama and Loomis, 1974). However, the ultimate weakness of this approach is revealed in the context of the more general problem of event recognition. Researchers have claimed that the rigid motion assumption is required for unique interpretation of flow patterns because nonrigid motions allow an indefinite number of interpretations in terms of depth and motions (e.g., Hildreth and Hollerbach, 1987; Nayakama and Loomis, 1974). However, "nonrigid" has been used here incorrectly to mean "arbitrary" motion. Nonrigid motions are not arbitrary, as shown by the number of distinct kinds of "nonrigid" events that are recognizable.

In fact, the majority of studies demonstrating and investigating visual event recognition have involved nonrigid motions (Bingham, Rosenblum, and Schmidt, in press; Cutting, 1982; Fiandt and Gibson, 1959; Jansson and Johansson, 1973; Jansson and Runeson, 1977; Todd, 1982), and in particular those of human actions (Barclay, Cutting, and Kozlowsky, 1978; Cutting, 1978; Cutting and Kozlowsky, 1977; Cutting, Proffitt, and Kozlowsky, 1978; Frykholm, 1983; Johansson, 1973, 1976; Todd, 1983). These studies alone, however, do not reflect the proportion or variety of recognizable events involving different kinds of nonrigid motions. Such motions include varieties of bending, as of a human trunk or elbow, a paper clip or a tree limb buried in snow; types of folding, tearing, and crumpling, as of pieces of paper, the body of a car, or a loaf of fresh Italian bread; varieties of breaking, as of glass, a cookie, a wooden board, or a loaf of stale Italian bread; types of elastic stretching or compressing, as of a hair net, a bouncing ball, a tree branch blowing in the wind, vibrating jello, or a human face forming various expressions; kinds of plastic deformations, as in forming clay figures, kneading bread, making snowballs, or leaving footprints in soil; types of liquid flows

involving the pouring, running, bubbling, and splashing of liquids of varying viscosity, as of water, oil, molasses, or thickening gravy cooking on the stove; varieties of flows of gases, as of steam or smoke in air; snow or leaves blown in a breeze, and so on. The great diversity of different types of non-rigid events that might be perceptually identified renders any simple distinction between rigid and nonrigid far too weak and inadequate to address the problem of visual event identification.

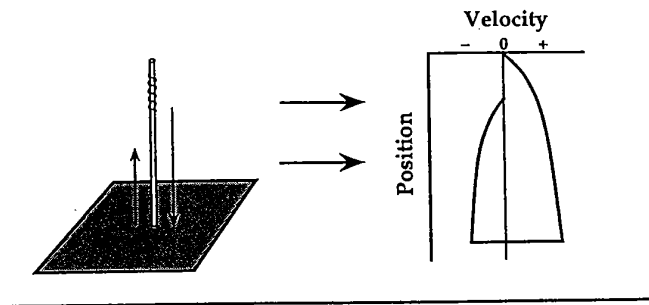
The rigidity of objects is a physical property which, like elasticity, plasticity, or fluidity, can generate specific types of motions. The question is whether observers are able to recognize such properties in specific instances and if so, how? More generally, the identification problem is, first, to discover what types of events and event properties observers are able to recognize and, second, to describe the information enabling them to do so. For instance, Bingham et al. (in press) have shown that observers were able to recognize events including free fall and elastic rebound, swinging pendulums, rolling balls, stirred water, objects dropped into water, and tickets of paper blown and falling through air, all from the forms of motion displayed in patch-light video recordings.

The patch-light technique isolates motion as information from static figural properties. Events are filmed so that bright patches of reflective material placed on surfaces in events appear against a dark (structureless) background. When these displays are freeze-framed, they appear as only a random array of irregular patches. When set in motion, the recorded events are typically recognized quite readily.

In the Bingham et al. study, observers' descriptions of the patch-light events reflected the underlying types of dynamics rather than simple kinematic similarities like the presence or absence of rotational motion in the display. Events involving rigid-body dynamics were described as more similar to one another and distinguished from hydrodynamic or aerodynamic events which were similarly grouped. Observers also distinguished the inanimate motion of a falling and bouncing object from the animate motions produced when the same object was moved by hand along the same path, to the same endpoints, and at the same frequency. Motions produced by the biodynamics reflected increases in mechanical energy, while those produced only by rigid-body dynamics reflected strict dissipation of energy. In all cases, recognizably different events were produced by different generative dynamics.

The forms of motion corresponding to each event were sampled from the video recordings and captured in phase-space trajectories. In each case, the trajectory form reflected the dynamics that generated the form. For instance, as shown in figure 14.1, the free fall and bounce produced a parabolic trajectory (characteristic of gravity) with a flat base (corresponding to the impact and elastic rebound) followed by a decelerative parabolic trajectory rising to a height diminished by energy dissipation. In contrast, the object moved by

Free Falling and Bouncing Spring



Spring Moved by Hand

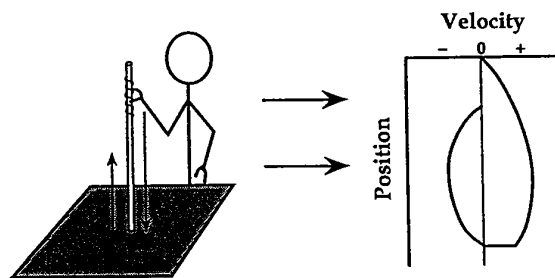


Figure 14.1 (Top) The phase trajectory of a free-falling and bouncing spring. (Bottom) The phase trajectory of the same spring moved by hand to the same endpoints at the same frequency.

hand produced an elliptical trajectory (characteristic of human limb movement) with a half-flat base (corresponding to inelastic impact and loss of energy), followed by an accelerative elliptical trajectory (which reflected energy increase). These spatiotemporal forms in optical flows provided visual information enabling observers to recognize the corresponding events. Such information is paradigmatic of the understanding of perceptual information developed by Gibson.

14.3 DIRECT PERCEPTION: INFORMATION AND INFORMATIONAL BASES

How can optical patterns have perceptual significance? How can they provide information about objects and events in the surroundings? How can they specify what is happening? Two classic solutions to these questions were rejected by Gibson (Gibson, 1950, 1966, 1979; Reed, 1988; Reed and Jones, 1982). The first, usually attributed to Berkeley, is that optical patterns gain significance by virtue of associations with haptic experience, i.e., touch and kinesthesia. The difficulty with this idea arises with the realization that haptics only functions well in the context of voluntary movements. Objects

and properties of objects (e.g., surface compliance, surface texture, weight and inertial distribution, shape, etc.) can be identified rapidly and reliably only when an observer is allowed to actively explore and manipulate an object (Gibson, 1962, 1966; Klatzky, Lederman, and Metzger, 1985; Lederman and Klatzky, 1987). Understanding how spatiotemporal patterns of tissue deformation provide information about objects and events (including the perceiver's own activity) is, if anything, a more difficult problem than that encountered in vision. This is, in part, because the problems in understanding the control and coordination of actions are inherited as part of the problem of understanding haptics (although ultimately action is a part of the problem of visual recognition as well) (Bingham, 1988). More to the point, the effective patterns of tissue deformation that impinge on the sensory receptors in haptics are less accessible to measurement and manipulation in experiments. Finally, and most important, it is spatiotemporal patterns of tissue deformation, i.e., change in geometric configurations over time, that provide information in haptics just as in vision (Bingham, Schmidt, and Rosenblum, 1989; Pagano and Turvey, 1993; Solomon, 1988). This realization undercuts any intuition that a solution to problems in vision, if seemingly insoluble, should be found only in haptics.

The second classic solution is that optical patterns have significance by virtue of a similarity relation to that about which they provide information, i.e., that optical patterns are copies of environmental patterns. Gibson also rejected this alternative. Gibson's analysis of optical occlusion is a paradigmatic case (Gibson, 1979; Gibson, Kaplan, Reynolds, et al., 1969). The deletion of optical elements along a boundary specifies one surface becoming hidden by another by virtue of a change in perspective. With progressive deletion, optical elements cease to exist in the optical pattern. However, the significance of this optical flow pattern does not inhere in a similarity relation to what is specified. The optical pattern does not specify surface elements going out of existence in the environment. Why not? Because surfaces do not go out of existence neatly and quietly at an edge, although they do go out of existence in a variety of other ways constrained and determined by natural laws. Surfaces can burn, explode, evaporate, melt, break, and so on. Each of these types of events produces corresponding types of optical transformations that are distinct from progressive deletion along a boundary. Also, each of the former events is irreversible, whereas the hiding of a surface via change in perspective is reversible, yielding accretion of optical elements at a boundary. Thus, Gibson argued that the particular pattern of optical flow can specify an event to which it corresponds by virtue of natural laws that determine the particular form of both the event and the optical flow.

The historical precedents to this understanding take us back at least as far as Hume (1739/1978). He argued that perception only has access to motions, not causes, because optical (or acoustical, etc.) patterns involve space and time, but not mass or force. His skeptical argument was a natural extension of arguments to the effect that perception only has (direct) access to "phenomena" described via only two spatial dimensions and time because the

third spatial dimension is absent in optical pattern. Such phenomenalism has been standard fare in the philosophy of perception and widely advocated despite its leading inevitably to the absurdities of solipsism. Rejecting phenomenalism requires that perception have direct access to information specifying substantial properties of the surroundings (Shaw, Turvey, and Mace, 1981; Turvey, Shaw, Mace, et al., 1981).

Hume, writing just after the publication of Newton's *Principia*, resorted to the billiard table to illustrate his understanding. Hume recognized the invariance of the motions that ensue once the cue ball is sent rolling so as to collide with the eight ball. The same motions result each time the balls are positioned and set moving in the same way. Nevertheless, Hume argued that an observer could not obtain epistemological access to the substantial properties of the event because the latter lay beyond the mere motions and only the motions are communicated to the eye via patterns in light. Because of the unique relation between motions and their causes, the two cannot be separated and observers have no means by which to get past the kinematics to reach the dynamics. He argued that the observer has direct access only to patterns of contiguity in space and time.

Two hundred years later, Michotte (1963) performed demonstrations which contradicted Hume's conclusions. The irony is that Michotte used technology that was available to Hume so that Hume might have made the discovery himself. Michotte devised a way to simulate linear collisions in displays that enabled him to perturb the kinematics without concern for underlying dynamics. (See Michotte, 1963, for details. This is now achieved using computer simulations.) When shown Michotte's collision displays, observers recognized them as collisions. In these displays, one simulated object approached a stationary object, contacted it, and stopped moving, while the contacted object instantly carried on the motion. Michotte then inserted a brief delay at the point when the two simulated objects came into contact so that the second object hesitated for fractions of a second before beginning to move. The result was that observers no longer recognized the display as of a collision. The slight perturbation changed the perceptual significance. The implication was that particular kinematic patterns have particular perceptual significance.

The upshot was that Hume's argument should be turned on its head. Indeed, causal constraints on events produce invariant forms of motion given invariant initial conditions. The invariance is a reflection of the underlying determinism which allows motions (and corresponding optical patterns) to be informative. They are informative by virtue of unique correspondence. The correspondence is enforced by natural laws, i.e., by dynamics. Note that not just any kinematic pattern will be perceptually significant. The perturbed kinematics in Michotte's demonstration were rather odd. Forced to describe what they perceived, observers rather creatively described the display as specifying a "triggering" event, as if the first object triggered the release of energy stored in a spring which then sent the second object on its

way. However, the instantaneous acceleration of the second object does not look exactly like such a triggering event. Runeson (1977) pointed out that Michotte did not manipulate simulated dynamics to produce his displays and thus the simulations were inaccurate and the displays rather ambiguous.

Todd (1982) inadvertently illustrated this methodological difficulty by manipulating only kinematics while trying to discover the forms of motion specific to bipedal walking and running as recognizable types of locomotor events. Todd began with digitized trajectories from actual walking and running. He then independently manipulated the motions for each of the 7 degrees of freedom in his stick figure legs, mixing and matching motions from the walking and running. The results were always rather ambiguous. Some looked more like running or walking as the case might be, but none were very convincing. Todd concluded that he really did not have a clue as to the essential characteristics of motions identifiable as walking or running and that he was lost in the sea of kinematic possibilities allowed by the 7-degrees-of-freedom system. A better approach would be to start from an understanding of the dynamics of these locomotor events. Walking can be understood as a system of upright and inverted pendulums, whereas running entails a mass-spring dynamic (McMahon, 1984). Investigation should proceed via perturbations performed with respect to these dynamics. For instance, as implied by the pendular dynamics and as shown by Bingham et al. (in press), the orientation of the kinematics in the gravitational field contributes to its perceptual significance. Perturbation of the orientation alters the significance. Likewise, would recognition be stable in the face of perturbation of the gravitational value or the stiffness of the leg or changes that alter the direction of energy flows among the link segments?

Runeson and Frykholm (1983) formulated kinematic specification of dynamics (or KSD) as a principle to be used to guide investigations of perceptual information. They referred to dynamics as an "informational basis," meaning that which enabled kinematic pattern to specify events. In so doing, they made explicit what remained implicit in Gibson's original analysis of occlusion. Gibson was circumspect about the importation of dynamics to the study of perception (Gibson, 1979; Reed, 1988). He emphatically wished to avoid the "error of confusing descriptions with things described" (so named by Dewey and Bentley, 1949). Gibson referred to perceptible properties as "affordances" to keep them well anchored within a functional context in which perception is motivated by action. As such, perceptible properties remain to be discovered and described by perception research. They cannot not be found in a dictionary of physics or in Webster's. Nevertheless, the powerful analytical apparatus of dynamics can be applied to analysis and perturbation of optical information as long as we remain mindful of the fact that types of recognizable events and event properties need not correspond directly to any of the familiar concepts in dynamics (Bingham and Muchisky, 1993a,b; Bingham et al., in press).

An essential aspect of this approach is the realization that unique correspondence between, e.g., optical pattern and events can only be found at certain levels of analysis. The scope must be sufficiently wide to include relevant structure. For instance, no single momentary velocity in a collision is specific to the type of event (anymore than it would be sufficient for a dynamical analysis of the event). Rather, the pattern of variation in velocities over *significant spatial and temporal extents* is required to achieve unique correspondence. This point is central to the current discussion of event recognition and the critique of extant analyses of optical flow. In the "structure from motion" corpus, analysis of optical flow has been restricted to structure captured in brief moments spanning a sampling interval of a few milliseconds, namely, a vector field. Such structure could not be used to identify events because, as shown by Bingham et al. (in press) and related studies, events are only specified by structure in optical flow that emerges over the entire course of an event. The information must be contained in various forms of optical transformation occurring at specific rates corresponding to the rate structure of motions in an event.

The mapping of event motions into optical flows can be described in terms of a relation between kinematic variables. Event velocities at given positions in three-dimensional space project to optical velocities at corresponding positions in the two-dimensional optical array. The question is whether qualitative properties of event trajectories are preserved in the mapping to optical trajectories? The first difficulty is entailed by the projection from three- to two-dimensional space (or if we include velocities associated with each position coordinate, the projection from six- to four-dimensional space). Components of the event kinematics that are radially directed with respect to the point of observation do not map directly into optical flows. Nevertheless, as will be shown, radial components of event kinematics do determine distinguishable components of optical flow that preserve the original forms of the event kinematics.

The final difficulty underlying the problem of event recognition is the degrees-of-freedom problem, that is, the problem of reducing the complex and large number of distinct motions (e.g., of patches) to the simple coherent motion of a single event. The optical phase space mapped from an event will contain an extremely large number of distinct trajectories. Any event consists of a continuous spatial distribution of points, each following a different trajectory. Only portions of the original event kinematics find their way into the optics. During an event, points go out of and come into view as they are occluded by other parts of a moving object or by surrounding objects. This happens not only with a rolling ball but also as limbs appear and disappear behind one another when a person locomotes or when a tree blows in the wind. It occurs as waves on the ocean occlude one another and passing vessels, as cars occlude one another in traffic, or as pedestrians occlude one another on city sidewalks, as a dancer performs pirouettes, as one stirs one's oatmeal, and so on. The result is that most any given trajectory is sliced into

myriad disjoint pieces which, together with those from other trajectories, produce a massive collection of nonidentical trajectory pieces. The disjoint character of the pieces from a given trajectory coupled with the simultaneous presence of arbitrarily close pieces from distinct trajectories prevents the simple reconstruction of individual trajectories. Given such a tangled mass of trajectory pieces, how might a single, unitary, and coherent event be apprehended? I will demonstrate how symmetries apparent in the layout of optical trajectories can allow the dimensionality to be reduced so that the underlying form might be apprehended. The suggested bottom line is that the global structure of event trajectories is required to yield a specification of events.

14.4 THE RELATION OF EVENT KINEMATICS TO DYNAMICS: THE KINEMATIC SPECIFICATION OF EVENTS

Rejecting the "Missing-Dimension" Characterization

Originally, the formulation of the kinematic specification of dynamics was inspired by the ability of observers of patch-light displays to apprehend values of dynamic properties such as relative mass or lifted weight (Bingham, 1987b, 1993; Runeson, 1977; Runeson and Frykholm, 1981, 1983; Runeson and Vedeler, 1993; Todd and Warren, 1982). In the context of this scaling problem, kinematic specification of dynamics has been cast as an "inverse dynamics" problem. Inverse dynamics, or the derivation of dynamics from kinematics, has been described, in turn, as a missing-dimension problem (Bingham, 1987b; Runeson and Frykholm, 1983; Warren and Shaw, 1985). Kinematic variables (e.g., position, velocity, acceleration, etc.) are defined using only the length and time dimensions [L, T].³ For instance, position might be expressed in meters (dimensionally [L]) and velocity in meters per second (dimensionally [L/T]). On the other hand, dynamic variables (e.g., mass, force, stiffness, damping, etc.) also require the mass dimension [M]. So, mass might be expressed in kilograms (dimensionally [M]) and force in kilogram-meters per second squared (dimensionally [ML/T²]). For inverse dynamics, how is the missing mass dimension recovered from kinematics?

For instance, the dynamic equation describing a mass-spring oscillator is $m(d^2x/dt^2) = -kx$, where m is mass, k is stiffness, x is position, and d^2x/dt^2 is acceleration. The terms in this equation involve [M] because each includes a mass-related (i.e., dynamic) parameter, namely m or k , as well as kinematic variables x or d^2x/dt^2 . Dimensionally, m and k are [M] and [M/T²] while x and d^2x/dt^2 are [L] and [L/T²], so each term in the equation is dimensionally a force, i.e., [ML/T²]. The dynamic equation determines motions or behaviors described via a kinematic solution equation, in this case $x = A \sin(\omega t + \phi)$. In this equation, the amplitude, A , and the phase, ϕ , are kinematic constants that depend only on initial conditions. Thus, they are arbitrary in respect to the dynamics. In contrast, the angular frequency, ω , is determined by the two dynamic parameters, $\omega = (k/m)$.⁵ In this ratio, the mass dimension cancels

out leaving a quantity that is kinematic ($([M/T^2]/[M])^5 = [T^{-1}]$), and thus appropriate for the kinematic equation. However, because of this, the kinematics, used as information about the dynamics, can only yield a determination of the ratio of the dynamic parameters, k/m . Determination of unique values of either m or k is not possible. This is a typical instance of the missing-dimension problem of inverse dynamics.

A potentially general solution to this problem is revealed by studying a closely related type of dynamic, that of the simple pendulum. The key to the solution is the observation that a unique value for one parameter would be specified if *identifiable circumstances* constrained the value of the remaining parameter. The dynamic equation for the simple pendulum can be written as $(d^2\phi/dt^2) = (g/l)\sin\phi$, where ϕ is the angular position at the pivot, l is the length of the pendulum, and g is the gravitational acceleration. The situation seems the same as for the mass-spring system because the frequency of motion corresponds to a ratio of parameters, $\omega = (g/l)^{1/2}$. However, g is a scaling invariant in the terrestrial sphere. Gravitationally determined trajectories appear as parabolas in phase space. By virtue of this characteristic form, gravitational trajectories can be recognized, in principle. This particular circumstance, or "uniquity condition" (Szűcs, 1980), imposes an identifiable scaling constraint so that the frequency of motion (or its inverse, period) specifies the length of the pendulum. Indeed, as shown by Pittenger (1985, 1990), observers are able to evaluate pendulum lengths on the basis of periods of oscillation. Uniquity conditions may provide a general means by which scaling problems are solved (Bingham, 1988). If so, then the particular circumstances that determine a scaling constraint must be identifiable. This is a second way in which scaling would entail recognition.

At this point, the reader might have noted that there was no mass parameter in the pendulum equation. Dimensionally, the equation that I used was kinematic. g is an acceleration $[L/T^2]$ while l is a length $[L]$. However, following the Newtonian procedure of "force balance," the equation would first have been written as $ml^2(d^2\phi/dt^2) = mgl\sin\phi$, where $ml^2 = I$ is the rotational inertia. When the inertia is divided into both sides of the equation, the mass cancels and the terms in the resulting equation have kinematic dimensions only. This trick is not peculiar to the pendulum. For instance, the dynamic equation used to describe the mass-spring oscillator can also be written as $d^2x/dt^2 = -(k/m)x$, which has the dimensions of acceleration $[L/T^2]$. Nor does the elimination of the mass dimension mean that the "dynamics" were eliminated. To the contrary, the dynamics are simply those lawful relations that generate specific forms of behavior given initial (and boundary) conditions.

The fact that this strategy is general and paradigmatic⁴ suggests that the "missing-dimension" characterization of the kinematic specification of dynamics is misleading. The problem is not to recover a missing mass dimension so much as to recognize a particular (perhaps scale-specific) type of event generated by a configuration of scaling parameters on kinematic variables.

Ultimately, dynamic equations can always be formulated in a dimensionless form in which no units are associated with any of the terms in the equation. A dimensionless equation is achieved by forming ratios among the elements in an equation so that the associated units cancel, leaving pure numbers (Baker, Westine, and Dodge, 1973; Emori and Schuring, 1977; Szücs, 1980; Thompson and Stewart, 1986). For instance, the equation for a force-driven damped mass-spring oscillator is as follows:

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F \cos \omega t \quad (1)$$

Each term in this equation has the dimensions of force, $[ML/T^2]$. So dimensionally, the equation is:

$$[M] \left[\frac{L}{T^2} \right] + \left[\frac{M}{T} \right] \left[\frac{L}{T} \right] + \left[\frac{M}{T^2} \right] [L] = \left[\frac{ML}{T^2} \right] \left[\frac{T}{T} \right]$$

To write equation (1) in dimensionless form, one can formulate a set of dimensionless numbers (sometimes called pi numbers (Emori and Schuring, 1977)) in terms of ratios and products of the original set of parameters and variables.

$$\begin{aligned} \pi_1 &= \frac{x}{x_0} \stackrel{d}{=} \frac{[L]}{[L]} & \pi_2 &= \omega t \stackrel{d}{=} \frac{[T]}{[T]} & \pi_3 &= \frac{bt}{m} \stackrel{d}{=} \frac{\left[\frac{M}{T} \right] [T]}{[M]} \\ \pi_4 &= \frac{kt}{b} \stackrel{d}{=} \frac{\left[\frac{M}{T^2} \right] [T]}{\left[\frac{M}{T} \right]} & \pi_5 &= \frac{mF}{b^2 x_0} \stackrel{d}{=} \frac{[M] \left[\frac{ML}{T^2} \right]}{\left[\frac{M}{T} \right]^2 [L]} \end{aligned}$$

where x_0 is a reference length, such as the undeformed length of the spring. Next, one can write the original equation in terms of these dimensionless variables as follows:

$$\frac{d^2\pi_1}{d\pi_3^2} + \frac{d\pi_1}{d\pi_3} + \frac{\pi_1 \pi_4}{\pi_3} = \pi_5 \cos \pi_2 \quad (2)$$

Equations (1) and (2) are analytically equivalent. When the parameters and variables in equation (2) take on values, they are pure numbers with no associated dimensions and the same is true of the solution equation which would be of the form:

$$\pi_1 = f(\pi_2, \pi_3, \pi_4, \pi_5).$$

See Szücs (1980, pp. 275–279) for additional discussion of this example and other techniques for achieving the same results. See also Baker et al. (1973, pp. 22–29). A closely related example can be found in Thompson and Stewart (1986, pp. 292–294) who used forcing frequency and amplitude to scale a forced oscillator system in dimensionless form. In this latter case, if the forcing were treated as a control function, then the behavior of the oscillator

would be scaled intrinsically in terms of the controls. In any of these cases, dynamics and kinematics are made dimensionally commensurate because dimensions are removed and no missing-dimension problem exists. Nevertheless, the scaling problem remains because the scale values of dimensionless parameters are still at issue.

Essentially, one tailors an equation to express the dynamics most efficiently by placing an equation in the appropriate dimensionless form so that each dimensionless parameter is directly responsible for a particular aspect of the resulting behavior. Thus, the values of dimensionless parameters determine the specific behavior exhibited by a differential equation, especially in the case of nonlinear dynamics (Hirsch and Smale, 1974; Rosenberg, 1977; Thompson and Stewart, 1986). In principle, the dynamics can be arranged to exhibit scale invariance, that is, a lack of change in the form of behavior despite a change in scale. The dimensionless parameter values and associated behavior can be preserved over scale changes by affecting scale changes in the original dimensional parameters of proportionate degrees determined by the ratios in the dimensionless form of the equations (Mandelbrot, 1983; Schroeder, 1991; Thompson and Stewart, 1986). In the forced mass-spring example, as b , the damping, is changed, one would alter m , the mass, proportionately so as to preserve the value of π_3 and thus maintain the form of the behavior exhibited by the system. Of course, k and F would also have to be altered to preserve the values of π_4 and π_5 , respectively.

The problem in the majority of actual events, as known all too well by scale engineers (Baker, Westine, and Dodge, 1973; Emori and Schuring, 1977), is that scale values along various dimensions cannot be arbitrarily varied. The values are associated with specific materials. Some values may occasionally be altered by substituting one material for another; however, a material usually determines, not just one, but a collection of relevant values along different dimensions (Baker, Westine, and Dodge, 1973, pp. 312-322; Emori and Schuring, 1977). So, a scale engineer will typically test a single functional property in a small-scale model that distorts other functionally important properties of the full-scale ship, airplane, or dam. In actual events, all of the ratios in an equation can be preserved over scale changes only in rare instances, and strictly never. The implication is that specific forms of motion are associated with particular types of events occurring at specific scales. This is why the small-scale models used to film disasters (e.g., collapsing dams, bridges, or buildings) in grade B science fiction films are usually quite obvious. Merely filming the small-scale event at high speed and scaling down the time leaves the trajectory forms unchanged and those forms are distorted in the small-scale event.

If the type of event can be recognized via the form of the behavior, then the scaling associated with relevant dynamic parameters might be determined. Generally, only the values of dimensionless parameters might be specified by trajectory forms. However, if recognizable circumstances (e.g., gravity, air resistance) were to constrain the values of dimensional parameters

within the dimensionless ratios, then values of other dimensional parameters might also be determined.

I have shown in this section that the kinematic specification of dynamics is not a missing-dimension problem. The missing-dimension characterization is a form of dualism that would render kinematics and dynamics as fundamentally incommensurate aspects of an event. Ultimately, this would make kinematic specification of dynamics or of events impossible (not to mention mechanics itself). Both dynamics and kinematics can be expressed in dimensionless equations. Thus, they are entirely commensurate. Dimensions are relevant, nevertheless, to the formulation of a dynamical model. Dimensions are a necessary part of the bookkeeping required to proceed from law forms and extrinsic measurement procedures to a dynamical model of an event. But it is a mistake to reify such dimensional notation as fundamental ontological types. The so-called fundamental dimensions (i.e., mass, length, and time) are not fundamental. In mechanics, dimensional analysis requires three dimensions, but the particular dimensions vary in different formulations (Duncan, 1953; Ipsen, 1960; Langhaar, 1951; Sedov, 1959). The more productive focus in trying to understand the relation between kinematics and dynamics is on the (abstract) form of events. To anticipate, kinematics corresponds, in this qualitative perspective, to particular trajectory forms, whereas dynamics yields an entire family of trajectories. Kinematics is relatively local, whereas dynamics is relatively global.

Understanding Event Perception via the Qualitative Approach to Dynamics

In the qualitative approach to nonlinear dynamics, both dynamics and kinematics are construed geometrically as alternative, and therefore commensurate, descriptions of common underlying forms (Marmo, Saletan, Simoni, et al., 1985; Thompson and Stewart, 1986). The forms are described in terms of vector fields from the perspective of dynamics, whereas from a kinematic perspective they are described in terms of trajectories. The dynamic vectors are tangent to the kinematic trajectories at all points.

This qualitative characterization is both the more elegant and the more appropriate for two reasons at least.⁵ First, a dynamic is determined by the form of the vector field or the trajectories. A dynamic cannot be identified with particular equations used to describe it because many different equations can be used to describe the same dynamic depending on the type of coordinates (Hirsch and Smale, 1974; Marmo et al., 1985). The form of the vector field or the corresponding phase-space trajectories remains the same despite change in coordinates.

Second, a qualitative construal of dynamics is the most natural given our intended application in event perception (Bingham, 1987a; Kugler, 1983). This, given as a reason for a qualitative interpretation, might seem rather circular in this context. However, given the fact that observers do perceive

events (i.e., what has sometimes been called an "existence proof"), together with the fact that dynamic factors determine kinematic forms of motion and that kinematics must provide the information allowing events to be recognized, then there must be a commensurate or symmetry relation between kinematics and dynamics. To the extent that dynamic types and perceived types of events correspond, the mapping between kinematics and dynamics must be invertible and by definition there can be no missing-dimension problem. The only possible solution is that provided by the qualitative interpretation.

Events as Dynamics Coupled with Uniqueness Conditions

In the linear tradition, dynamics as such is distinguished from specification of the range of potential parameter values and other "uniqueness conditions" (Szűcs, 1980). The goal in dynamics has been to generalize across events involving different types of objects or materials. Uniqueness conditions, namely parameter values as well as initial and boundary conditions, must be specified before solutions to linear dynamics can be derived. These uniqueness conditions have been held separate from the dynamic itself because they necessarily restrict the generality of the description. However, the relation between parameter values and dynamics is not so dissociable in nonlinear dynamics because the specific forms of behavior are closely tied to the values of the parameters. With the recognition that dynamics must be identified not with equations, but with forms of behavior, uniqueness conditions become an integral part of the dynamics.

Only by tying strongly restricted ranges of parameter values and other uniqueness conditions to a given dynamic can we establish a correspondence between perceptually recognizable types of events and dynamics. This means that the formal character of dynamics must be enlarged to incorporate mathematical apparatus that has not been included in the dynamics of the linear tradition. Dynamical systems theory is based on the operations of the calculus which become undefined at discontinuities in trajectories (Hirsch and Smale, 1974; Tufillaro, Abbott, and Reilly, 1992). Some discontinuities are merely effects of the scale of measurement and can be handled by appropriately upgrading the (nonlinear) dynamics at the appropriate scale. For instance, when differential equations have been used to describe the damping of motion in an event involving viscous or kinetic friction, actual cessation of motion has occurred only in infinite time where the trajectory finally asymptotes at zero velocity. In actual events, motion ceases in relatively brief finite time as friction transits from kinetic to static form or as a lubricating substance becomes adhesive at low shear velocities. Nevertheless, improved models could capture such transitions as highly nonlinear forms of damping.

On the other hand, discontinuities are also produced by impacts and contact between object surfaces. These are extremely common in daily events.

The problem in this case is that the relative location of surfaces is contingent, not determinate. Once the contingencies are established, an event does unfold in a determinate fashion that reflects the particular contingencies. Such contingencies are unicity conditions. To the extent that they are specified in the behavior of a system, they must be included in its dynamics. This entails two modifications in dynamical systems models of actually perceived events (Bingham, 1990). First, piecewise continuous dynamical systems are required and second, some form of Boolean logic will have to be integrated with the calculus of smooth dynamical systems. Boolean logic is a formal means of handling contingencies as conditionals. For instance, modeling the trajectories in a free fall and bouncing event requires a projectile motion dynamic during one position-dependent portion of the trajectory and an extremely stiff mass-spring oscillatory dynamic during another. (For another example, see Thompson and Stewart, 1986, pp. 291–320.) The forms of the trajectories that result are specific both to the nature of the event as a free fall and bounce and to the contingent height of the object above the surface with which it collides (Muchisky and Bingham, 1992). Using Boolean logic, one tests state variables (i.e., positions or velocities) to determine when trajectories have entered regions of the state space that have been assigned different smooth dynamics.

There are two unicity conditions that are universal in the terrestrial sphere but that are not usually tied to dynamics. One is temporal while the other is spatial. Dynamics is generally isotropic with respect to both time and space. The anisotropy (or irreversibility) of time is a well-recognized problem in the context of linear dynamics (Prigogine, 1980; Prigogine and Stengers, 1984). Nevertheless, for the vast majority of perceptible events, identity is not preserved over time reversal as revealed by Gibson in his study of reversible vs. irreversible events (Gibson and Kaushall, 1973). In such instances, the positive sign on the time variable must be preserved as a unicity condition. In the terrestrial sphere, dynamics is also spatially anisotropic because gravity contributes as a dynamic scale factor to the form of all perceptible events (including the activity of potential observers). Gravity establishes a definite orientation in the terrestrial domain reflected in the resulting asymmetric forms of events. Objects fall downward and roll down hills to come to rest in valleys. Bingham et al. (in press) found that the perceived significance of many kinematic forms changed when, unknown to observers, the orientation with respect to gravity of event kinematics in displays was changed. Clearly in such cases, both the sign and the value of gravity must be included as unicity conditions intrinsic to the event dynamics. In general, any factor that contributes to a determination of kinematic forms and the perceived significance of those forms must be included in the dynamics used to model an event.

The final type of unicity condition that must be included as an inherent part of dynamic event models are initial or boundary conditions. These are values that determine transient states or trajectories. The focus of study in the qualitative approach to nonlinear dynamics is usually on stable trajectories (or

attractors).⁶ Stable behavior corresponds to phase trajectories that do not change radically in form with small changes in parameter values or in initial conditions. With the appropriate changes in parameters, however, the trajectories will exhibit a bifurcation, that is, a rapid transition to a new stable behavior.

In the study of event perception, the forms of interest must include those of transient trajectories as much as, or more than, those of stable trajectories. All inanimate events are damped and so ultimately cycle down to point attractors. However, once the attractor has been reached, the event is over. The most informative states are the transients yielding optical flow. Examples of such transients would be a branch set oscillating by the wind, a ball that has been dropped and bounces or rolls downhill, a coin that has been dropped and rolls and rattles to a stop, and finally, a swing oscillating until coming to rest after having been abandoned by a child.

The forms of trajectories, transient and stable alike, can be partially classified according to attractor states. But as indicated by the examples, a more specific classification (including perhaps the metric forms of trajectories) will be required in event perception. Exactly what level of scaling (i.e., ratio, interval, ordinal, etc.) will be required to capture the relevant qualitative characteristics of kinematic forms depends both on a theoretical determination of the qualitative properties of event kinematics that are preserved in the mapping to optical flows and on the empirical determination of the detectable qualitative properties of optical flow (e.g., Norman and Todd, 1992).

14.5 MAPPING FROM EVENT KINEMATICS TO OPTICAL FLOWS

If qualitative properties of an event trajectory are to provide information allowing event identification, then those properties must map into optical flows. What are the qualitative properties that map into optical flows? Certain properties are bound to be preserved. For instance, discontinuities corresponding to impacts will map to discontinuities in optical flows. Periodic events will map to periodic optical flows. However, other properties will be lost. For instance, event trajectories exhibiting different conic sections (i.e., an elliptical curve vs. a parabola) are confused in optical flow. Spatial metrics are lost because trajectories are scaled by viewing distance in the mapping to optical flows. This scale transformation induces changes in trajectory shapes because the scaling variable (i.e., distance) is itself a kinematic variable and not a constant. However (assuming an immobile point of observation) the course of values of the scaling variable is phase-locked to the remaining kinematic variables so that *the forms in optical phase space are related to those in event phase space by projective transformations, just as the forms of objects are related to their imaged forms*. The mapping of forms in phase space is essentially the same as the mapping of three-dimensional object forms because the metric structure of the time dimension in events is preserved while the spatial metric is lost in the same way as for objects.

To illustrate the projection of event trajectory forms into patterns of optical flow, I did a simulation of a ball rolling along a U-shaped groove from its release until it nearly stopped moving following appropriate laws of motion. The ball was inelastic, 0.27 m in diameter, and weighed 1 kg. It rolled without slipping along an inelastic U-shaped surface that was 1.0 m high and 2.5 m wide. The event included many aspects typical of rigid-body events, including translation along a constraint surface accomplished via rotations; harmonic motion associated with the gravitational potential; and dissipation associated with air resistance and surface friction. Together, these produce kinematics typical of a damped harmonic oscillator, as shown in figure 14.2. (This can be shown analytically; e.g., see Becker, 1954, pp. 206–207.)

Motion was confined to a vertical X-Y plane. However, the plane of motion did not lie perpendicular to the visual direction,⁷ so this case was sufficiently general. The perspective was from directly above the event looking down, so that significant components of motion occurred both parallel and

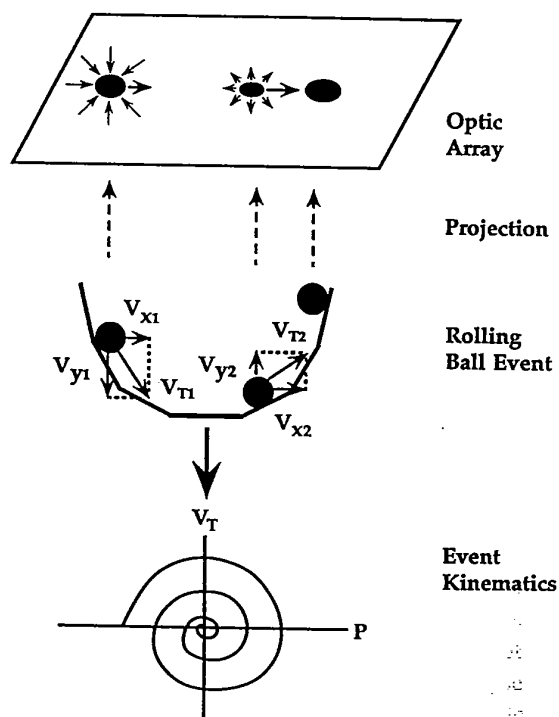


Figure 14.2 The kinematics of the rolling ball event and their projection into optical flow. Event kinematics: A plot of the ball tangential velocity, V_T , against the ball position along the surface yielded a spiral as the ball's oscillation back and forth damped out. Projection to optics: The X component of V_T , mapped into a component common to all optical points projected from the ball, represented by a single vector in the projection plane. The Y component of V_T , mapped into radial inflow or outflow (i.e., "relative motion") as the ball moved away from or toward the point of observation, respectively.

perpendicular to the visual direction. A rigid-body analysis was employed. Accordingly, translatory and rotational components of motion were separated. Each of these components map respectively into common and relative components in optical flow (see figure 14.2). Johansson (1950, 1973, 1976) has shown that the visual system behaves as if decomposing optical flow into these components. (Subsequently, I discuss the difficulties associated with an inability to assume rigid-body motion.)

Using orthographic projection, as shown in figure 14.2, the tangential velocity of the ball, V_T , maps into the optics via two components, V_X and V_Y . V_X , perpendicular to the visual direction, maps into the optics as a vector common to all points from the ball. V_Y , parallel to the visual direction, maps to a set of vectors organized in a radial inflow or outflow pattern depending on the momentary approach or retreat of the ball, respectively. Thus, the event kinematics map via components into very different aspects of the optical flow.

I simulated the event from the dynamics using numerical methods. The simulated event duration was 12 seconds. A smoothly curved U-shaped surface was approximated via a set of nine contiguous linear segments. Discontinuities that appeared in the resulting trajectories reflected small collisions at transition points between successive segments.

Kinematics of the Center of Mass: Common Translatory Motion

The motion of the center of mass corresponds to translatory motion common to every point in the ball. We reduced the dimensionality of the translatory event kinematics from four dimensions (X, Y, V_x , and V_y) to three by using the tangential velocity, $[V_x^2 + V_y^2]^{.5} = V_T$. These kinematics are depicted in figure 14.3 where the trajectory through a phase space consisting of the X and Y position and the tangential velocity, V_T is plotted together with the X and Y components projected orthographically on the $X-V_x$ and $Y-V_y$ planes respectively. The U-shaped path was also projected on the $X-Y$ plane.

The problem was mapping the forms on the $X-V_x$ and $Y-V_y$ planes into the optics from a perspective at some distance above the event (i.e., in the Y direction). The $X-V_x$ component mapped via a single scaling factor, the viewing distance Y , into a single optical component common to all points from the ball (i.e., divide each by Y). As shown in figure 14.4A, this component carried the essential spiraling damped oscillator form of the original trajectory. However, based on this alone, the event could not be distinguished from horizontal, planar motion produced by an oscillating air hockey puck attached to a spring. The $Y-V_y$ component was essential to completely capture the translatory event structure. This is an important point because in recent reviews research on motion parallel to a projection plane has been reviewed separately from research on perpendicular motion with the corresponding implication that the two components are functionally distinct, the former being used for event perception and the latter for visually guided

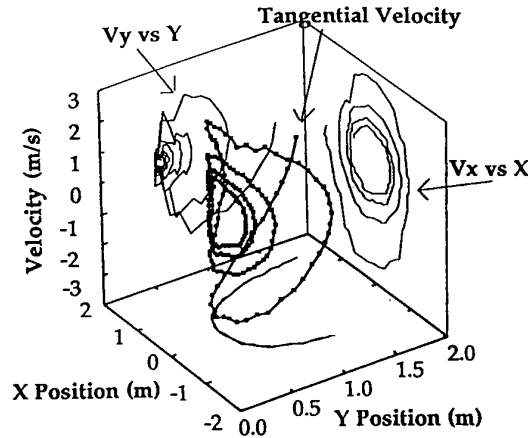


Figure 14.3 The tangential velocity trajectory of the ball plotted against X-Y position, with the corresponding path projected on the X-Y plane. The V_y -Y and V_x -X components were projected onto the respective planes.

activity (Anstis, 1986; Regan, Kaufman, and Lincoln, 1986). Clearly, this cannot be the case.

Next I examined the projection of the Y-V_y component. This did not map directly into a single common component of the optical flow. Using a linear approximation appropriate for the viewing distance, the form on the Y-V_y plane could be mapped, via a single scale factor Y, into optical variables detectable by human observers, namely image size and the rate of expansion (or contraction) of the image. To achieve this mapping, I divided object radius, r, by viewing distance, Y, yielding image size. Taking the derivative of image size, I computed image expansion rate as rV_y/Y^2 . As shown in figure 14.4B, a plot of rate of expansion vs. image size preserved the form of the Y-V_y phase portrait.⁸

The structure carried into the optics along the Y component was more easily interpreted in a plot of expansion rate vs. X/Y. The latter is the optical correlate of X position. This plot reproduced the form of an X-V_y plot. Trajectories that were successively interspliced in figures 14.3 and 14.4B were unfolded in figure 14.4C, revealing more plainly the trajectory that resulted as the ball rolled up one side of the U, stopped, and rolled down again, reaching zero V_y at the bottom of the U, rolling up the other side of the U to a stop, and so on. The phase-locked relation between the X and V_y components enabled this plot and reflected the fact that a perspective on a single three-dimensional form (a single trajectory in X-Y-V_y phase space) was being described.

To summarize, forms associated with both X and Y components of the center of mass trajectory were mapped successfully into forms associated with detectable optical flow variables. Both components of motion were required to specify the translatory motion of the ball.

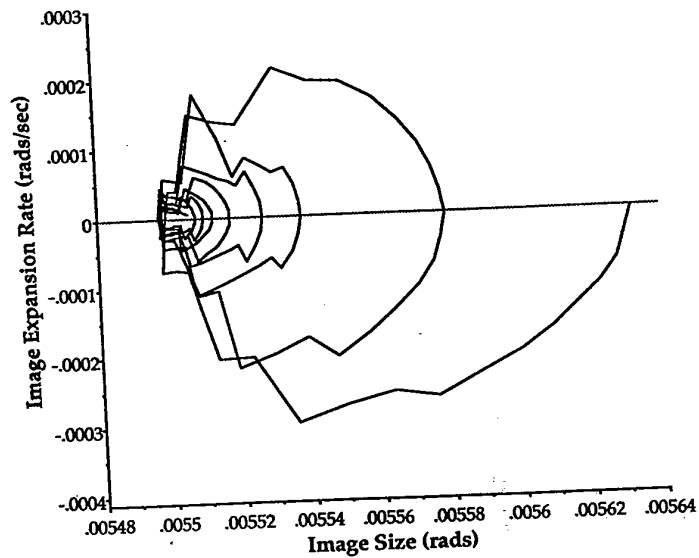
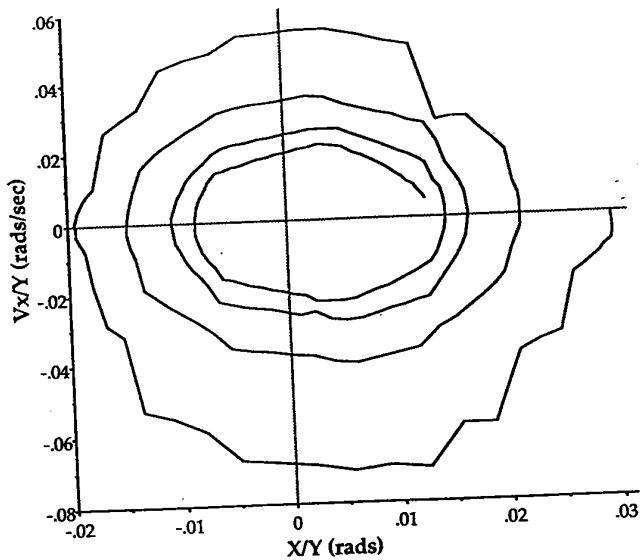


Figure 14.4 (A) The V_x - X component mapped into corresponding optical variables via parallel projection. (B) The V_y - Y component mapped into corresponding optical variables via parallel projection. (C) A plot of V_y vs. X mapped into corresponding optical variables.

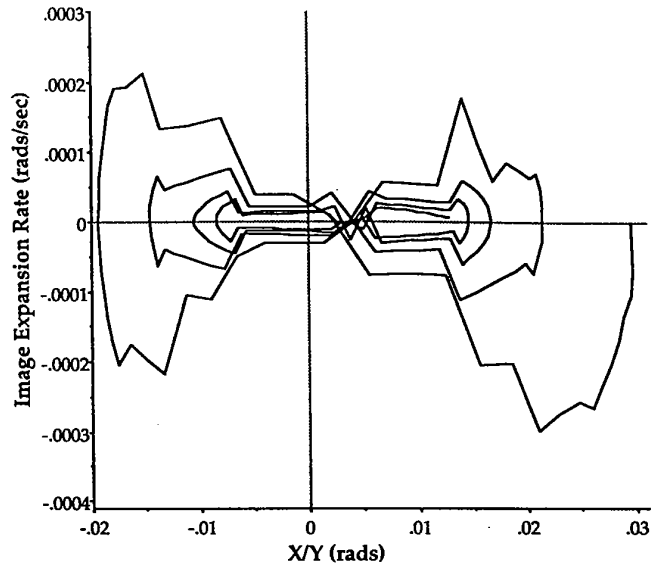


Figure 14.4 (cont.)

Rotational Motion About the Center of Mass

Next, the rotational motion of the ball must be considered. To start simply, the rotational kinematics will be examined within a frame of reference fixed in the ball, ignoring the translatory motion of the ball. In figure 14.5A, the ball is shown side-on, looking along its axis of rotation. Over the course of the event, the angular velocity of the ball about this axis, V_a , varied exactly as did the velocity of the center of mass, V_T . Multiplying V_a by the perpendicular distance, L , from the axis of rotation to the ball surface yielded V_L , the instantaneous linear velocity of corresponding points on the ball surface. This linear velocity vector was of constant magnitude for each point about the axis of rotation within any plane parallel to the plane of motion, as shown in figure 14.5A. Within the plane of motion through the center of the ball, L was equal to the radius of the ball and V_L was equal at each moment to V_T . Moving out of this plane along the ball surface toward the point on the side where the axis of rotation pierced the ball surface, V_L shrank to zero as did L , as shown in figure 14.5B.

As shown in figure 14.5C, when the frame of reference was changed from the (moving) ball to the (fixed) constraint surface, V_T was brought back into consideration and V_L was added to V_T at each point on the ball because V_T was common to all points on the ball. The result was that the ball axis of rotation moved at V_T while the part of the ball in contact with the constraint surface was (momentarily) at rest ($-V_L + V_T = 0$) and the top of the ball opposite this contact point moved at $2V_T$ (because $V_L = V_T$).

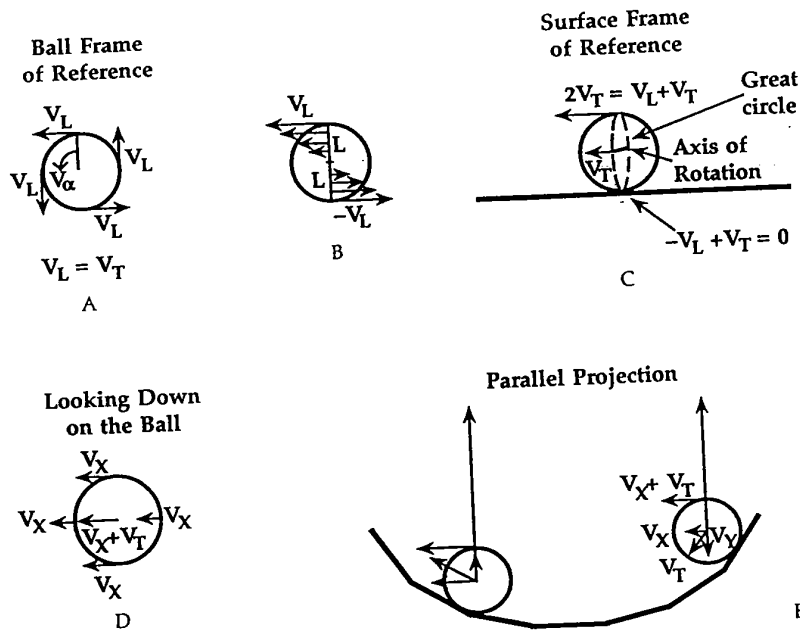


Figure 14.5 (A) Ball rotation represented in a frame of reference fixed in the ball. V_T is removed. V_α is the rotational velocity; L is the distance from the axis of rotation to a given point on the ball surface; V_L is the tangential velocity at a point on the ball surface. For points around the midline of the ball, $V_L = V_T$. (B) Variation in V_L moving along the ball surface from the midline to the point where the axis of rotation pierces the surface. (C) Ball motion represented in a frame of reference fixed in the constraint surface on which it rolls. The axis of rotation translates at V_T . The tangential velocity at the top of the ball is $2V_T$ while at the bottom point in contact with the constraint surface, it is 0. (D) Ball motion looking straight down on the ball. At the contour, tangent velocity from rotational motion is pointed directly toward or away from the point of observation and so is lost. Only the V_x component of V_T remains. (E) The components for parallel projection into optical flow.

Three points in this kinematic structure are uniquely identifiable. The point on the ball surface pierced by the axis of rotation is identifiable as the center of a vortex of velocities. This point has no rotational component of motion, only the translatory component, V_T .

The points on the top and bottom of the ball are also unique points identifiable as the points of maximum and minimum velocity respectively. These points and the gradient of velocities running between them provide information about the location of the constraint surface relative to the ball. Along a great circle formed by intersecting the ball surface with a plane perpendicular to the direction of V_T and containing the axis of rotation, velocities would follow a gradient from zero at the point of contact with the constraint surface to $2V_T$ at the opposite point. Viewing the event from a distance directly above, the velocity of all points along the contour of the ball's image would be the V_x component of V_T as shown in figure 14.5D, whereas points in

the interior of the ball's image would follow a gradient up to a velocity of $V_X + V_T$ at the center of the image.

The rate structures associated with the rotational motion of the ball are qualitatively equivalent to those associated with the translatory motion of the center of mass of the ball. Each of the velocities within the gradient along the ball surface project to corresponding X and Y components depending on perspective. These components would follow phase-space trajectories identical in form to those described above for X and Y components of V_T . For instance, viewing the event from a distance directly above, the velocity components at the center of the ball's image would be $V_X + V_T$ and V_Y , respectively, as shown in figure 14.5D and E. The form of the rate structure associated with the $V_X + V_T$ component is qualitatively equivalent to that of the X- V_X plot.

To summarize, qualitative properties of the rate structure of this event mapped successfully into optical flows via both the rotational and translatory components of the ball's motion.

As will be shown subsequently, the spatial gradient of flow vectors associated with the rotational motion provides information about the shape of the ball (Todd and Reichel, 1989; Todd and Akerstrom, 1987). This spatial gradient *in the context of the rate structure* also provides information about the ball's relation to the constraint surface, the surface lying at the momentary point of zero flow. The orientation of the constraint surface, in turn, provides information about the direction of gravity which corresponds to the direction of the constraint surface at moments when V_T reaches its relative maxima along the trajectory. For the ball to roll without slipping, the constraint surface must always lie below the center of mass of the ball. That it does so is specified by the way the projected velocities vary along the constraint surface.

14.5 THE DEGREES-OF-FREEDOM PROBLEM IN VISUAL EVENT PERCEPTION

Nearly all extant analyses of "structure from motion" use the rigidity assumption (e.g., Andersen, 1990; Horn, 1986; Koenderink, 1986; Koenderink and van Doorn, 1975, 1976, 1987; Lee, 1974, 1980; Longuet-Higgins and Prazdny, 1980; Nakayama and Loomis, 1974; Owen, 1990; Rieger, 1983; Rieger and Lawton, 1985; Ullman, 1984; Warren, 1990; Waxman and Ullman, 1985; Zacharias, 1990). The rigidity assumption has been used because it drastically reduces the degrees of freedom in optical flow. Using results from analytical mechanics (Rosenberg, 1977; Whittaker, 1944), the motion of a rigid body can be described in terms of the translation of its center of mass combined with a rotation around that center. Alternatively, translation and rotation relative to the point of observation can be used. In either case, the positional degrees of freedom of the three-dimensional motion are reduced from $3n$ degrees of freedom, where n is the number of distinguishable points in the body, to 6 degrees of freedom, 3 to specify the position of the

center of mass and 3 to describe the body's orientation about its center. In mechanics, additional degrees of freedom are required to specify a body's state of motion. The velocities (but only the velocities) corresponding to each of the positional degrees of freedom must also be specified at some time, t_0 . When these are specified together with a dynamic, the subsequent motion of the object is determined.

Ultimately, however, the rigidity assumption is untenable because it requires that an observer know in advance what he or she is perceiving to be able to perceive, i.e., a rigid-body event. This is an obvious paradox. Alternatively, the assumption restricts the relevant models to an unrealistically small set of perceivable situations, excluding any sort of nonrigid event. On the other hand, without the rigid-body assumption, the degrees of freedom required to specify the state in an event is $6n$, i.e., 3 positions and 3 velocities for each distinguishable point. Depending on how one distinguishes points on an object surface (with the projection to optics in mind), this number grows indefinitely large fast. Furthermore, the problem projects right into the optics despite both the loss of points via occlusion by opaque surfaces and the reduction to a total of four coordinates for each point in the optical flow (2 positions and 2 corresponding velocities).

The nature and severity of this problem will be conveyed by returning to the rolling ball example. The kinematics of the event were described in spherical coordinates with the origin fixed at an unmoving point of observation located about 2.5 m from the event. The trajectories of a mere 12 points on the surface of the ball were selected for study, 4 points at 90-degree intervals around the ball in each of three planes parallel to the plane of motion, one plane at the center coincident with the plane of motion and one plane to either side at 70% of the distance from the center to the side of the ball. In other respects, the simulation was the same as described earlier, including the duration, which was 12 seconds. The resulting event trajectories were projected into optical flow.

The optical flow trajectories were captured in a three-dimensional optical phase space by using θ and ϕ position coordinates together with the tangential velocity to the optical path or orbit. θ and ϕ are visual angles in a polar projection appropriate for viewing at nearer distances. Only components perpendicular to the visual direction at each point in the event projected into the optics, each scaled by the distance along the visual direction. However, as the ball rolled, each point on its surface successively went out of view as it rolled underneath the ball and then into view as it rolled over the top of the ball. The result was that only discontinuous pieces of trajectories appeared in the optical flow, including only those portions of the trajectories that were not occluded by the ball itself. The optical phase portrait appears in figure 14.6.

If we were able to count only single trajectories associated with each of the 12 points on the ball, then the number of degrees of freedom would be 12×4 coordinates = 48. However, as can be seen in figure 14.6, it is not obvious how the various trajectory pieces go together. The trajectories are

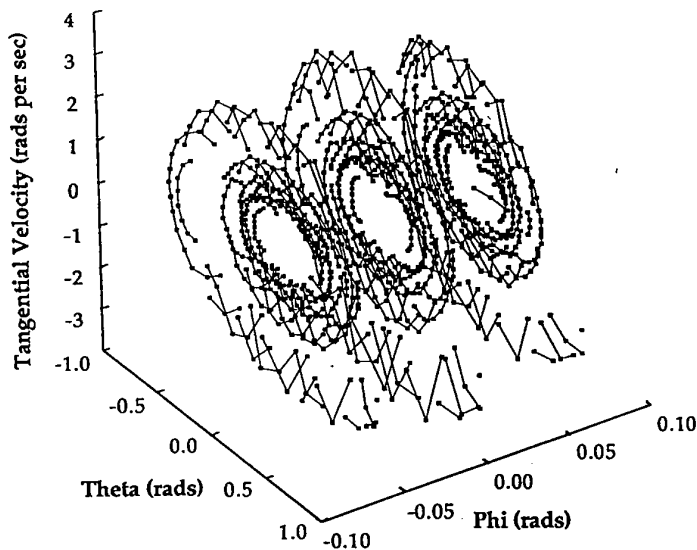


Figure 14.6 An optical phase-space portrait of the ball rolling event. Trajectories correspond to 12 points around the ball surface, 4 around the midline, and 4 around a point to either side of the midline. Tangential optical velocities plotted against two optical (angular) position coordinates, theta and phi. The phi-axis was rescaled relative to the theta-axis to better reveal the structure in the trajectories.

not piecewise continuous. The pieces are separated by significant intervals. Thus, the degrees of freedom in optical phase space had better be enumerated by counting the degrees of freedom associated with each trajectory piece. The 12 points moving over 12 seconds yielded 244 trajectory pieces, each requiring ultimately 4 coordinates which I reduced to 3 using the tangential velocity, with $244 \times 3 = 732$ degrees of freedom resulting! This was from a mere 12 points on the ball. The distinguishable optical texture elements on such a surface could easily yield more than 1000 points resulting in over 45,000 degrees of freedom.

Solving the Degrees-of-Freedom Problem via Symmetries in Optical Phase Space

Recall the description of the rolling ball example with which I began this chapter. The observer is confronted with a random array of moving patches each of which appears, moves a modest distance in the display, and then disappears (perhaps never to be seen again). Despite the apparent complexity and difficulties of this display, observers immediately perceive the display as of a rolling ball. How is this possible? With the denial of the rigid-motion assumption, we arrive at the unmitigated problem of visual event identification. How is the tremendous number of degrees of freedom associated with

the trajectories in optical flow reduced to the relatively few degrees of freedom associated with the coherent and well-formed motions in recognizable events? The effect of occlusion combines with the degrees-of-freedom problem to exacerbate the problem by orders of magnitude. Resort must be made to time-extended samples of optical flow to find enough structure to solve the identification problem. (Time-extended trajectories also yield stability of the optical structure in response to perturbation by noisy measurements.) The strategy will be to find symmetries among the trajectories in the phase plane portrait and to use them effectively to collapse the structure, reducing the degrees of freedom and, at the same time, obtaining coherence and revealing the underlying form.⁹

A glance at figure 14.6 reveals that the phase trajectories contain a number of symmetries (i.e., commonalities of form) that might be used to reduce the degrees of freedom in the optical flow. For instance, the spiral on the phase plane, characteristic of damped oscillatory events, can be seen in common across the three sampled planes of motion, although this form becomes rather lost among the overlapping trajectory pieces past the first cycle. In an earlier section of this chapter, the optical flow from the rolling ball was described using properties such as the contour of the ball's image and the centroid of the image. The advantage in deriving trajectories from these image properties was that the issue of occlusion was avoided, i.e., the resulting trajectories were continuous.

To illustrate this, the flow at 5 points in the ball's image was computed including the centroid as well as the front, back, and opposite side points on the contour relative to the common direction of motion. The resulting optical trajectories were plotted in figure 14.6 where the spiraling forms of the trajectories could be seen much more clearly, as could the symmetries among the trajectories. The continuous trajectories in figure 14.7 certainly represent a reduction in the degrees of freedom from those in figure 14.6.

Note that in a patch-light display there is no closed, continuous contour forming the boundary of the ball's image. There is only a random array of moving patches yielding trajectories, as in figure 14.6. The event is nevertheless identifiable. The question, therefore, is how might we derive the coherent trajectories in figure 14.7 from those in figure 14.6? To solve this question, we need to examine the structure of the trajectories appearing in figure 14.6 more closely. Figure 14.8A shows the trajectories projected from the 4 points around the middle of the ball. The highly structured character of the phase portrait is quite apparent in this figure. Each separate trajectory piece or hoop corresponds to the motion of a single point on the ball as it rises up from the back over the top of the ball and disappears in the front. The variations in θ distances between the ends of each hoop in turn correspond to the variations in image size. The rounded form of the hoops is related to the rotation of the ball. The rounded trajectory form is created as the rotational velocity component is progressively added in and then removed as a point travels from the back over the top to the front of the ball. This first symmetry is common to

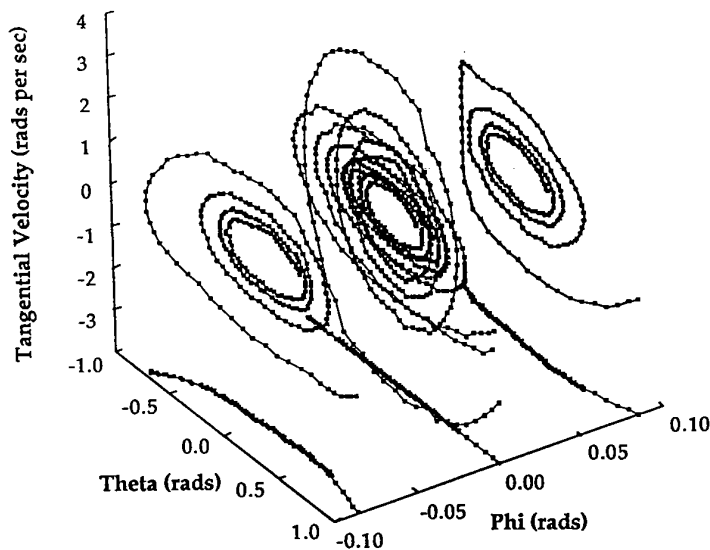
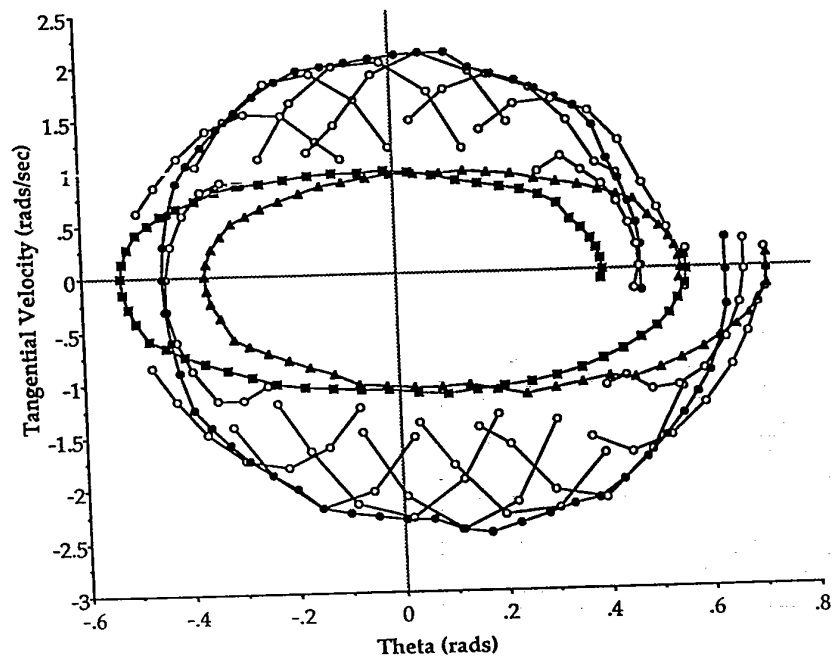
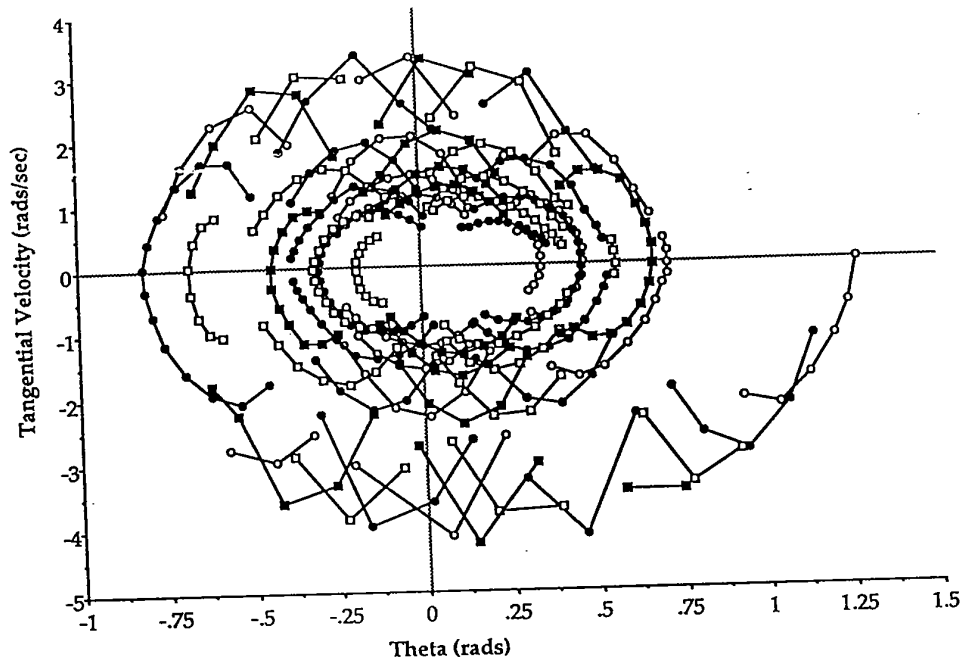


Figure 14.7 An optical phase portrait derived by tracking 4 points on the ball image contour plus the point in the center of the image. The points on the contour were points front and back on the midline and side points farthest from the midline. The paths of motion of these points were projected onto the theta-phi plane.

the trajectories in every plane parallel to the plane of motion and will ultimately allow us to collapse the trajectories in all the planes down to those in one plane, for instance that in figure 14.8A. But first, we should analyze the structure in that plane.

The most important symmetry is the envelope surrounding the train of successive hoops. This provides the means of deriving the trajectories of figure 14.7 from those of figure 14.6. As can be seen in figure 14.8B, where I have plotted one cycle of the motion, the trajectories from figure 14.7 form the boundary on the envelope of trajectory pieces from figure 14.6. The bottom ends of the hoops correspond to the front and back occluding contours of the ball's image. The trajectories of these contour points are implicit, yet apparent in the flow from a mere 4 points. If the trajectories of more points were to be included, the contour trajectories would be more densely specified. The same is true of the image centroid, although in that case it is the apex of successive hoops that is involved.

An alternative and natural coordinate system in which to capture these trajectories is in terms of a phase angle and an energy-related measure of radial distance which I will call "energy." These are polar coordinates on the θ -by-tangential velocity plane (i.e., the plane in figure 14.8A and B) with the origin in the center of the spiral.¹⁰ Thus, these coordinates are intrinsic to the phase portrait. They are determined by landmarks on the trajectories themselves, namely, the points of peak and zero velocity. As implied by the



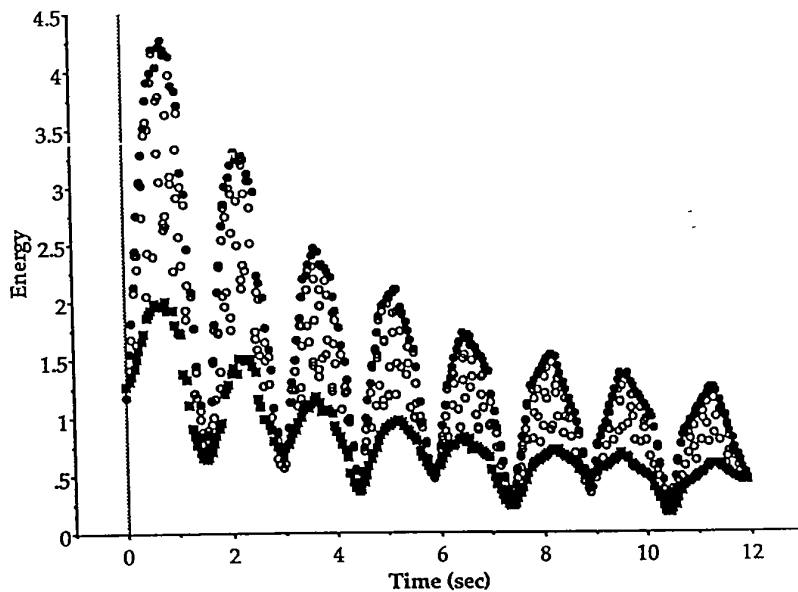


Figure 14.8 (A) Trajectories from 4 points around the midline of the ball. The 4 points in turn are represented by open circles, filled circles, open squares, and filled squares, respectively. Note that without these symbols, it would be impossible to determine which trajectory pieces represent a given point in common. (B) One cycle from the trajectories in figure 14.6A together (open circles) with midline trajectories from figure 14.5. The contour point on the front of the ball is represented by filled squares; the back of the ball by filled triangles, and the center of the ball image by filled circles. (C) The energies for all of the trajectories from figure 14.9A and B plotted against time. Open circles represent the 4 points tracked around the midline of the ball. Filled circles represent the center point of the ball image. Filled squares represent contour points.

coordinate labels, these coordinates also relate directly to the underlying dynamics.

When the trajectories in figure 14.8B were plotted in figure 14.8C as energy vs. time, the manner in which continuous trajectories bounded the envelope of the trajectory pieces could be seen quite clearly.

Returning to figure 14.6, I note that the properties revealed on the center plane obtained as well on the planes to the side. This suggests the solution to the next problem, which was to relate the motions on side planes to those on the center plane. The three sets of trajectories were 1:1 phase-locked. This could be seen by linearly regressing the phase angles (i.e., the first polar coordinate) for corresponding center and side points as "parameterized" by time. This is shown in figure 14.9A. The results were slopes near 1 (= .97 or better), intercepts near 0 ($\pm .002$ or less) and $r^2 = .999$ in all cases. The phase-locked relation between the center and side trajectories meant, given the symmetry of form, that I could collapse the different sets of trajectories

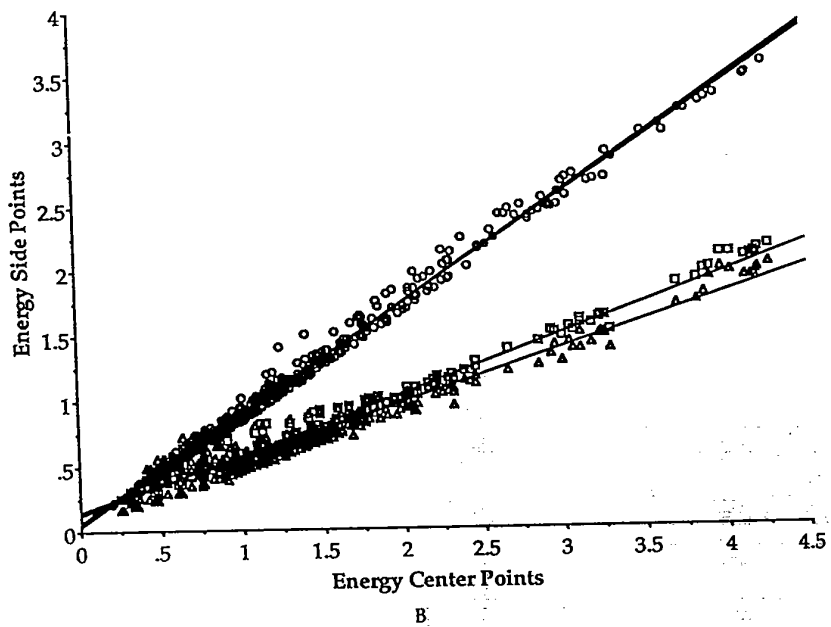
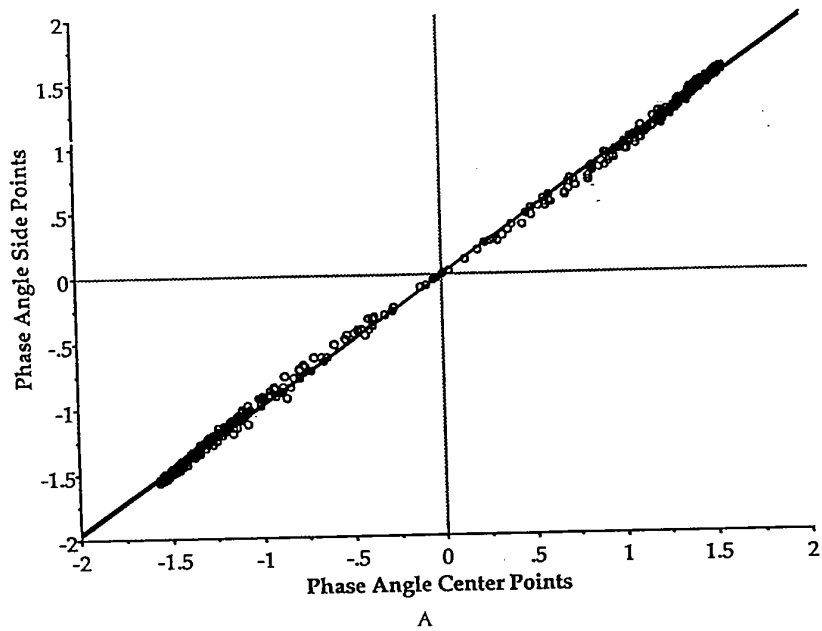
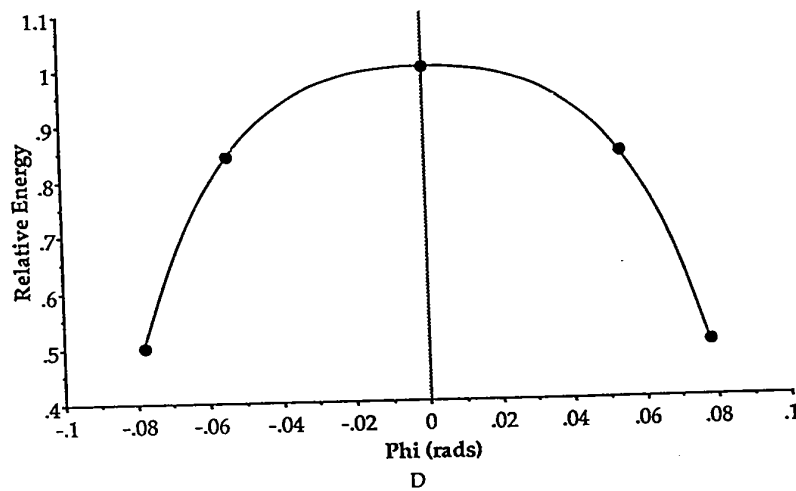
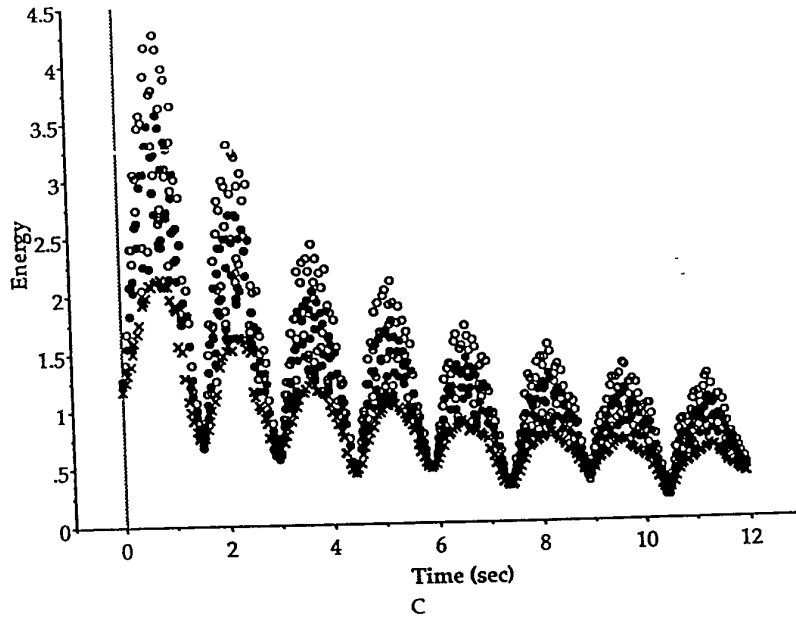


Figure 14.9 (A) Phase angle of the 4 points tracked about the ball midline linearly regressed on phase angles for the 4 points tracked about a line 70% of the distance from the midline to the side of the ball. (B) Energy of the 4 points tracked about the ball midline linearly regressed on the energies for the 8 points tracked about lines 70% of the distance from the midline to the



side of the ball (open circles); on the energies for the contour points, both front and back (open triangles) and side points (open squares). (C) Energies for the 4 points tracked about the midline (open circles), for the 4 points tracked about a line 70% of the distance to the side (filled circles), and for a contour point on the side (x's). (D) Slopes of the regressions in B plotted against the mean phi position for the corresponding points on the ball and fitted with a polynomial curve.

4.5

sed
to
sed
the

by normalizing to a common scale, e.g., rescaling by dividing in each case by the peak energy and reducing all trajectories to a common set with a peak energy of 1. These, in turn, could be reduced to trajectories of the same form as the center trajectories appearing in figure 14.7.

Of course, these symmetries of form also serve to make obvious the differences in scale among the sets of trajectories. This is important because the differences in energies (or radial lengths) is also informative. As is apparent in figure 14.7, the sizes of the spirals decrease from those corresponding to the middle of the ball to those at its sides. The relative heights of energies on the center plane, on the plane 70% of the distance to the side contour point, and at the side contour point appear in figure 14.9C plotted against time. The energy of the center points was linearly regressed on that for corresponding side points as parameterized by time, as well as on the energy for points on the contour at the back. The results are shown in figure 14.9B. Center point energy regressed on energy for side points 70% of the way to the outside edge of the ball yielded slopes of .84 with intercepts near 0. When center point energy was regressed on energy for the side point on the contour, the mean slope was .52. When center point energy was regressed on energy for the back point on the contour, the mean slope was .47. These results mean that if I assign 1.0 to the height of the trajectories along the middle of the ball, then the height of the trajectories 70% of the distance toward the sides is .84, while the height of the trajectories on the side contour is .52 (and on the back contour, .47). The relative heights of the middle trajectories, the side point trajectories, and the side contour point trajectories were plotted in figure 14.9D against their (mean) ϕ -coordinate values respectively and fitted with a polynomial curve. There one can see that these relative energies represent the shape of the ball.

Undoubtedly, I could find additional information in the qualitative properties of these trajectories with further analysis. These trajectories are replete with structure that can be discovered via processes sensitive to the symmetries among trajectory forms. Once discovered, the symmetries enable a reduction of the degrees of freedom and a recovery of coherent form which relates directly to the generative dynamics.

14.7 THE NECESSITY OF ANALYZING RATE STRUCTURES IN OPTICAL FLOW FOR EVENT RECOGNITION

Confronting the problems associated with the rate structures in events is not optional in the study of optical flow. By definition, information in optical flow is found in the time evolution of spatially distributed optical structure. One can represent the spatial distribution of the instantaneous values of rate variables as a vector field, but such a snapshot will fail to capture structure specific to given events.

The amount of information contained in the optical phase-space trajectories for the rolling ball should be compared to that contained in the instantaneous optical flow field analyzed in the majority of studies on optical flow.

Orbits corresponding to the middle and side point trajectories were projected on the theta-phi plane in figure 14.7. That these orbits in theta-phi configuration space contain considerably less information is rather apparent. The instantaneous optical flow field would correspond at some arbitrary moment to a set of 2 successive points along each orbit projected on the floor of figure 14.7 and the line drawn between the 2 points in each case. The result would be three very short line segments. This is not quite correct, however. The more appropriate projection would be from points along trajectories in figure 14.6. Also, a more dense set of points would be required than the 12 points represented in figure 14.6. Nevertheless, the character of the event could not be conveyed, no matter how dense the vector field.

Although some information about the shape of the ball can be gleaned from the instantaneous vector field (Koenderink and van Doorn, 1975, 1976, 1978; Waxman and Ullman, 1985) and the assumption of rigidity can often be checked (Longuet-Higgins and Prazdny, 1980), the nature of the event can only be apprehended from the information contained in time-extended trajectories. The spatial distribution in the optical flow corresponding to the rolling ball changed over time in nonarbitrary ways, such that any single sample in time of the spatial distribution could not be representative of the optical structure projected from the event. At endpoints of the trajectories, the ball momentarily stopped moving as it reversed direction and the optical flow field (instantaneously) ceased to exist. Along the trajectory, the point of maximum flow varied in its relative position within the contours of the ball's image. The flow field would not be strictly the same at any two points along the trajectory except at the two endpoints where the flow was null.

Rather than an insufficiency of structure, optical phase portraits contain an overabundance of structure that must be used to reduce the tremendous number of degrees of freedom associated with optical flows. The structure inheres in the forms of trajectory pieces and in symmetries existing across those forms. Of course, the symmetries or similarities of form must be noted to allow their use in reducing the degrees of freedom in optical flows. We have not ventured to describe processes instantiated in the sensory apparatus that would effect the measurements appropriate to uncovering symmetries and forms in optical phase space. Rather, by showing that the relevant properties of trajectories in events map into corresponding properties of optical trajectories and that such properties must be detected to recognize events, I have developed a job description for the sensory apparatus.

ACKNOWLEDGMENTS

This research was supported by National Science Foundation grant BNS-9020590. I express my gratitude to Esther Thelen, James Todd, Dennis Proffitt, and an anonymous reviewer for reading and commenting on an earlier version of this manuscript and acknowledge the assistance of Michael Stassen in deriving and performing the calculations reported in this chapter.

NOTES

1. Phase-space trajectories will not be sufficient for unique characterization of all events, some of which will require an event space, i.e., phase space plus a time axis (Rosenberg, 1977). Do not confuse this event space with an event phase space that is used in contrast to an optical phase space.
2. A symmetry is something that remains the same despite some change in other aspects of a form. For instance, the shape of a circle drawn on paper remains the same when the paper is translated along a table top. See Shaw, McIntyre, and Mace (1974) for a related discussion.
3. Dimensions appear in brackets in uppercase, whereas parameters or variables appear in lowercase.
4. The study of the pendulum was instrumental to the development of dynamics (Dijksterhuis, 1961; Jammer, 1957). Pendular motion was the core event investigated by dynamicists from Galileo, through Huygens (in particular), to Newton, and beyond. In modern nonlinear dynamics, the (force-driven) pendulum remains paradigmatic (Berge, Pomeau, and Vidal, 1984; Tufillaro, Abbott, and Reilly, 1992). In historical perspective, the dynamics of the pendulum is the epitome of dynamics.
5. Historically, it has been more productive to pursue dynamic properties as qualitative, rather than as material entities. The problem in construing dynamics in terms of material entities is the interpretation of forces. The essential nature of forces has been the subject of enduring debate in mechanics (Dijksterhuis, 1961; Jammer, 1957; Mach, 1893/1960). The most widely known phase of the debate involved the reaction to Newton's gravitational theory and "action at a distance." However, Galileo wrote, more than half a century earlier, of how he had elected to abandon attempts to describe the essence of gravitational action in favor of efforts to describe the form of trajectories in events involving gravity (Galileo, 1638/1914). The renowned result was the kinematics of free fall, recognized as among the most profound achievements in science. Newton's dynamics can be interpreted as having succeeded in generalizing Galileo's descriptions, enabling dynamicists to describe, among other things, free-fall trajectories on other planets as well as on earth (Jammer, 1957; Mach, 1893/1960). To this day, our understanding of gravity is couched in terms of geometry and the associated forms of trajectories (Taylor and Wheeler, 1966), although this interpretation remains controversial. The search for gravitational essence continues—e.g., cast as a search for gravity particles. Nevertheless, the historical precedents indicate that a focus on the form of trajectories has been a productive approach.
6. There are different types of stability (see, e.g., Thompson and Stewart, 1986). Structural stability refers to the stability of the form associated with stable trajectories as the dynamic equation is perturbed by adding a new term. A simple dynamic, which is not structurally stable, is the harmonic oscillator. This exhibits elliptical phase trajectories. When an arbitrarily small damping term is added to the equation, the trajectories change to spirals ending in a stable equilibrium state called a point attractor. Attractors are stable states such that all trajectories within a given neighborhood, the "basin of attraction," approach them and once near forever remain so unless perturbed violently enough to be forced from the basin. In the study of qualitative nonlinear dynamics, the forms of interest are the forms of the attractors, i.e., the (long-term) stable trajectories.
7. Releasing motion from the vertical plane would only introduce, in addition, an optical component corresponding to rotation about the visual direction. This has been described using differential operators as a curl (Koenderink and van Doorn, 1975, 1976).
8. Rather than the rate of image expansion and contraction, I also could have used $1/\tau$ or the inverse of tau (Lee, 1980). Computed as V_y/Y , this equals image expansion rate divided by image size. Plotted against image size, this also preserved the form of the $Y-V_y$ plot.

9. Todd (1982) developed a similar approach which he applied instead to the configuration space of paths or orbits. A similar use of symmetries in optics applied within a vector field representation can be found in Lappin (1990).

10. The phase angle is derived as the $\arctan(\text{tangential velocity}/\theta)$ while the length of the radius is $[(\text{tangential velocity})^2 + \theta^2]^{.5}$. The latter is related to the mechanical energy which equals the sum of the potential and the kinetic energy. We will call the coordinate "energy," although it is more properly related to the square root of the energy.

REFERENCES

- Andersen, G. J. (1990). Segregation of optic flow into object and self-motion components: foundations for a general model. In R. Warren and A. H. Wertheim (Eds.), *Perception and the control of self-motion* (pp. 127–141). Hillsdale, NJ: Erlbaum.
- Anstis, S. (1986). Motion perception in the frontal plane: sensory aspects. In K. R. Boff, L. Kaufman, and J. P. Thomas (Eds.), *Handbook of perception and human performance* (pp. 16:1–127). New York: Wiley.
- Baker, W. E., Westine, P. S., and Dodge, F. T. (1973). *Similarity methods in engineering dynamics: theory and practice of scale modeling*. Rochelle Park, NJ: Hayden Books.
- Barclay, C. D., Cutting, J. E., and Kozlowsky, L. T. (1978). Temporal and spatial factors in gait perception that influence gender recognition. *Perception and Psychophysics*, 23, 145–153.
- Becker, R. A. (1954). *Introduction to theoretical mechanics*. New York: McGraw-Hill.
- Berge, P., Pomeau, Y., and Vidal, C. (1984). *Order within chaos: toward a deterministic approach to turbulence*. New York: Wiley.
- Bertero, M., Poggio, T. A., and Torre, V. (1988). Ill-posed problems in early vision. *Proceedings of the IEEE*, 76(8), 869–889.
- Bingham, G. P. (1985). Kinematic form and scaling: further investigations on the visual perception of lifted weight. Doctoral Dissertation, University of Connecticut, Storrs, CT.
- Bingham, G. P. (1987a). Dynamical systems and event perception: a working paper, parts I–III. *Perception/Action Review*, 2(1), 4–14.
- Bingham, G. P. (1987b). Kinematic form and scaling: further investigations and the visual perception of lifted weight. *Journal of Experimental Psychology: Human Perception and Performance*, 13, 155–177.
- Bingham, G. P. (1988). Task-specific devices and the perceptual bottleneck. *Human Movement Sciences*, 7, 225–264.
- Bingham, G. P. (1990). Physical constraints on form: investigating visual information for event recognition and the judgment of size [unpublished].
- Bingham, G. P. (1993). Scaling judgments of lifted weight: lifter size and the role of the standard. *Ecological Psychology*, 5, 31–64.
- Bingham, G. P., and Muchisky, M. M. (1993a). Center of mass perception and inertial frames of reference. *Perception and Psychophysics*, 54, 617–632.
- Bingham, G. P., and Muchisky, M. M. (1993b). "Center of mass perception": affordances as dispositions determined by dynamics. In J. M. Flach, P. Hancock, J. Caird, et al. (Eds.) *The ecology of human-machine systems*. Hillsdale, NJ: Erlbaum.
- Bingham, G. P., Rosenblum, L. D., and Schmidt, R. C. (in press). Dynamics and the orientation

of kinematic forms in visual event recognition. *Journal of Experimental Psychology: Human Perception and Performance*.

Bingham, G. P., Schmidt, R. C., and Rosenblum, L. D. (1989). Hefting for a maximum distance throw: a smart perceptual mechanism. *Journal of Experimental Psychology: Human Perception and Performance*, 15, 507-528.

Cutting, J. E. (1978). Generation of synthetic male and female walkers through manipulation of a biomechanical invariant. *Perception*, 7, 393-405.

Cutting, J. E. (1982). Blowing in the wind: perceived structure of trees and bushes. *Cognition*, 12, 25-44.

Cutting, J. E., and Kozlowsky, L. T. (1977). Recognizing friends by their walk: gait perception without familiarity cues. *Bulletin of the Psychonomic Society*, 9, 333-356.

Cutting, J. E., Proffitt, D. R., and Kozlowsky, L. T. (1978). A biomechanical invariant for gait perception. *Journal of Experimental Psychology: Human Perception and Performance*, 4, 357-372.

Dewey, J., and Bentley, A. F. (1949). *Knowing and the known*. Boston: Beacon.

Dijksterhuis, E. J. (1961). *The mechanization of the world picture*. Princeton, NJ: Princeton University Press.

Duncan, W. J. (1953). *Physical similarity and dimensional analysis*. London: Edward Arnold.

Duncker, K. (1929/1950). Induced motion. In W. D. Ellis (Ed.), *A source book of Gestalt psychology* (pp. 161-172). New York: Humanities Press.

Eagleson, R. (1987). Estimating 3D motion parameters from the changing responses of 2D bandpass spatial frequency filters. In *IEEE Montreal Technologies Conference: Compint '87*, (pp. 102-105). Montreal: IEEE.

Emori, R. I., and Schuring, D. J. (1977). *Scale models in engineering: fundamentals and applications*. New York: Pergamon Press.

Fieandt, K., and Gibson, J. J. (1959). The sensitivity of the eye to two kinds of continuous transformation of shadow pattern. *Journal of Experimental Psychology*, 57, 344-347.

Frykholm, G. (1983). *Action, intention, gender, identity, perceived from body movement*. Uppsala, Sweden: University of Uppsala.

Galileo, G. (1638/1914). *Dialogues concerning two new sciences*. New York: Dover.

Gibson, J. J. (1950). *The perception of the visual world*. Boston: Houghton Mifflin.

Gibson, J. J. (1955). The optical expansion pattern in aerial locomotion. *American Journal of Psychology*, 68, 480-484.

Gibson, J. J. (1958). Visually controlled locomotion and visual orientation in animals. *British Journal of Psychology*, 49, 182-192.

Gibson, J. J. (1961). Ecological optics. *Vision Research*, 1, 253-262.

Gibson, J. J. (1962). Observations on active touch. *Psychological Review*, 69, 477-491.

Gibson, J. J. (1966). *The senses considered as perceptual systems*. Boston: Houghton Mifflin.

Gibson, J. J. (1979). *The ecological approach to visual perception*. Boston: Houghton Mifflin.

Gibson, E. J., Gibson, J. J., Smith, O. W., et al. (1959). Motion parallax as a determinant of perceived depth. *Journal of Experimental Psychology*, 58(1), 40-51.

Gibson, J. J., Kaplan, G., Reynolds, H., et al. (1969). The change from visible to invisible: A study of optical transitions. *Perception and Psychophysics*, 5, 113-116.

- Gibson, J. J., and Kaushall, P. (1973). *Reversible and irreversible events* (motion picture). State College, PA: Psychological Cinema Register.
- Hildreth, E. C. (1984). The computation of the velocity field. *Proceedings of the Royal Society of London, Series B*, 221, 189-220.
- Hildreth, E. C., and Grzywacz, N. M. (1986). The incremental rigidity scheme for recovering structure from motion: position versus velocity based information. In *Proceedings of the IEEE Computer Society Workshop on Motion* (pp. 137-143). IEEE Computer Society.
- Hildreth, E. C., and Hollerbach, J. M. (1987). The computational approach to vision and motor control. In F. Plum (ed.), *Handbook of Physiology* Vol. V, section 1. New York: Oxford University Press.
- Hildreth, E. C., and Koch, C. (1987). The analysis of visual motion: from computational theory to neuronal mechanisms. *Annual Review of Neuroscience*, 10, 477-533.
- Hirsch, M. W., and Smale, S. (1974). *Differential equations, dynamical systems, and linear algebra*. New York: Academic Press.
- Horn, B. K. P. (1986). *Robot vision*. Cambridge, MA: MIT Press.
- Hume, D. (1739/1978). *A treatise on human nature*. Oxford, England: Oxford University Press.
- Ipsen, E. C. (1960). *Units, dimensions and dimensionless numbers*. New York: McGraw-Hill.
- Jacobson, L., and Wechsler, H. (1987). Derivation of optical flow using a spatiotemporal-frequency approach. *Computer Vision, Graphics, and Image Processing*, 38, 29-65.
- Jammer, M. (1957). *Concepts of force: a study in the foundations of dynamics*. Cambridge, MA: Harvard University Press.
- Jansson, G., and Johansson, G. (1973). Visual perception of bending motion. *Perception*, 2, 321-326.
- Jansson, G., and Runeson, S. (1977). Perceived bending motions from a quadrangle changing form. *Perception*, 6, 595-600.
- Johansson, G. (1950). *Configurations in event perception*. Uppsala, Sweden: Almqvist & Wiksell.
- Johansson, G. (1964). Perception of motion and changing form. *Scandinavian Journal of Psychology*, 5, 181-208.
- Johansson, G. (1973). Visual perception of biological motion and a model for its analysis. *Perception and Psychophysics*, 14, 201-211.
- Johansson, G. (1976). Spatio-temporal differentiation and integration in visual motion perception. *Psychological Research*, 38, 379-393.
- Johansson, G. (1985). About visual event perception. In W. H. Warren and R. E. Shaw (Eds.), *Persistence and Change: Proceedings of the First International Conference on Event Perception*. Hillsdale, NJ: Erlbaum.
- Johansson, G., and Jansson, G. (1967). *The perception of free fall*. Unpublished report, Department of Psychology, University of Uppsala, Uppsala, Sweden.
- Klatzky, R., Lederman, S., and Metzger, V. (1985). Identifying objects by touch: an "expert system." *Perception and Psychophysics*, 37, 299-302.
- Koenderink, J. J. (1986). Optic flow. *Vision Research*, 26, 161-180.
- Koenderink, J. J., and van Doorn, A. J. (1975). Invariant properties of the motion parallax field due to the movement of rigid bodies relative to an observer. *Optica Acta*, 22(9), 773-791.
- Koenderink, J. J., and van Doorn, A. J. (1976). Local structure of movement parallax of the plane. *Journal of the Optical Society of America, Series A*, 66(7), 717-723.

- Koenderink, J. J., and van Doorn, A. J. (1978). How an ambulant observer can construct a model of the environment from the geometrical structure of the visual inflow. In G. Hauske and E. Butenandt (Eds.), *Kybernetik*, (pp. 224–247). Munich: Verlag.
- Koenderink, J. J., and van Doorn, A. J. (1987). The structure of the visual field. In W. Güttinger and G. Dangelmayr (Eds.), *The physics of structure formation: theory and simulation* (pp. 68–77). Berlin: Springer-Verlag.
- Kugler, P. N. (1983). *A morphological view of information for the self-assembly of rhythmic movement: a study in the similitude of natural law*. Ph.D. dissertation, Department of Psychology, University of Connecticut, Storrs.
- Langhaar, H. L. (1951). *Dimensional analysis and the theory of models*. New York: Wiley.
- Lappin, J. S. (1990). Perceiving the metric structure of environmental objects from motion, self-motion and stereopsis. In R. Warren and A. H. Wertheim (Eds.), *Perception and the control of self-motion* (pp. 541–579). Hillsdale, NJ: Erlbaum.
- Lederman, S., and Klatzky, R. (1987). Hand movements: a window into haptic object recognition. *Cognitive Psychology*, 19, 342–368.
- Lee, D. N. (1974). Visual information during locomotion. In R. B. McLeod and H. L. Pick (eds.), *Studies in perception: essays in honor of J. J. Gibson*. Ithaca, NY: Cornell University Press.
- Lee, D. N. (1980). The optic flow field: the foundation of vision. *Philosophical Transactions of the Royal Society London. Series B*, 290, 169–179.
- Lee, D. N., Young, D. S., Reddish, P. E., et al. (1983). Visual timing in hitting an accelerating ball. *Quarterly Journal of Experimental Psychology*, 35A, 333–346.
- Longuet-Higgins, H. C., and Prazdny, K. (1980). The interpretation of a moving retinal image. *Proceedings of the Royal Society London. Series B*, 208, 385–397.
- Mach, E. (1893/1960). *The science of mechanics: a critical and historical account of its development*. LaSalle, IL: Open Court.
- Mandelbrot, B. B. (1983). *The fractal geometry of nature*. New York: Freeman.
- Mark, L., Todd, J. T., and Shaw, R. (1981). The perception of growth: a geometric analysis of how different styles of change are distinguished. *Journal of Experimental Psychology: Human Perception and Performance*, 7, 355–368.
- Marmo, G., Saletan, E. J., Simoni, A., et al. (1985). *Dynamical systems: a differential geometric approach to symmetry and reduction*. New York: Wiley.
- Marr, D. (1982). *Vision*. San Francisco: Freeman.
- McMahon, T. A. (1984). *Muscles, reflexes, and locomotion*. Princeton, NJ: Princeton University Press.
- Michotte, A. (1963). *The perception of causality*. London: Methuen.
- Muchisky, M. M., and Bingham, G. P. (1992). Perceiving size in events via kinematic form. In J. Kruscke (Ed.), *Proceedings of the 14th Annual Conference of the Cognitive Science Society* (pp. 1002–1007). Hillsdale, NJ: Erlbaum.
- Nagel, H.-H. (1988). Image sequences—ten (octal) years—from phenomenology towards a theoretical foundation. *International Journal of Pattern Recognition and Artificial Intelligence*, 2(3), 459–483.
- Nakayama, K., and Loomis, J. M. (1974). Optical velocity patterns, velocity sensitive neurons, and space perception: a hypothesis. *Perception*, 3, 63–80.

- Norman, J. F. and Todd, J. T. (1992). The visual perception of 3-dimensional form. In G. A. Carpenter and S. Grossberg (Eds.), *Neural networks for vision and image processing*. Cambridge, MA: MIT Press.
- Owen, D. H. (1990). Perception and control of changes in self-motion: a functional approach to the study of information and skill. In R. Warren and A. H. Wertheim (Eds.), *Perception and the control of self-motion* (pp. 289-326). Hillsdale, NJ: Erlbaum.
- Pagano, C. C., and Turvey, M. T. (1993). Perceiving by dynamic touch the distances reachable with irregular objects. *Ecological Psychology*, 5, 125-151.
- Pittenger, J. B. (1985). Estimation of pendulum length from information in motion. *Perception*, 14, 247-256.
- Pittenger, J. B. (1990). Detection of violations of the law of pendulum motion: observers' sensitivity to the relation between period and length. *Ecological Psychology*, 2(1), 55-81.
- Pittenger, J. B., and Shaw, R. E. (1975). Aging faces as visco-elastic events: implications for a theory of nonrigid shape perception. *Journal of Experimental Psychology: Human Perception and Performance*, 1, 374-382.
- Prazdny, K. (1980). Egomotion and relative depth map from optical flow. *Biological Cybernetics*, 36, 87-102.
- Prigogine, I. (1980). *From being to becoming: time and complexity in the physical sciences*. San Francisco: Freeman.
- Prigogine, I., and Stengers, I. (1984). *Order out of chaos*. Toronto: Bantam Books.
- Proffitt, D. R., and Gilden, D. L. (1989). Understanding natural dynamics. *Journal of Experimental Psychology: Human Perception and Performance*, 15, 384-393.
- Reed, E., and Jones, R. (1982). *Reasons for realism: selected essays of James J. Gibson*. Hillsdale, NJ: Erlbaum.
- Reed, E. S. (1988). *James J. Gibson and the psychology of perception*. New Haven, CT: Yale University Press.
- Regan, D. M., Kaufman, L., and Lincoln, J. (1986). Motion in depth and visual acceleration. In K. R. Boff, L. Kaufman, and J. P. Thomas (Eds.), *Handbook of perception and performance: sensory processes and perception* (pp. 19/1-19/46). New York: Wiley.
- Rieger, J. H. (1983). Information in optical flows induced by curved paths of observation. *Journal of the Optical Society of America, Series A*, 73(3), 339-344.
- Rieger, J. H., and Lawton, D. T. (1985). Processing differential image motion. *Journal of the Optical Society of America, Series A*, 2(2), 354-359.
- Rosenberg, R. M. (1977). *Analytical dynamics of discrete systems*. New York: Plenum Press.
- Runeson, S. (1977). *On the visual perception of dynamic events*. Uppsala, Sweden: University of Uppsala.
- Runeson, S., and Frykholm, G. (1981). Visual perception of lifted weight. *Journal of Experimental Psychology: Human Perception and Performance*, 7, 733-740.
- Runeson, S., and Frykholm, G. (1983). Kinematic specification of dynamics as an informational basis for person and action perception: expectations, gender recognition, and deceptive intention. *Journal of Experimental Psychology: General*, 112, 585-615.
- Runeson, S., and Vedeler, D. (1993). The indispensability of precollision kinematics in the visual perception of relative mass. *Perception and Psychophysics*, 53, 617-632.
- Schroeder, M. (1991). *Fractals, chaos, power laws*. New York: Freeman.

- Sedov, L. I. (1959). *Similarity and dimensional methods in mechanics*. New York: Academic Press.
- Shaw, R. E., Mark, L. S., Jenkins, H., et al. (1982). A dynamic geometry for predicting growth of gross craniofacial morphology. In *Factors and mechanisms influencing bone growth* (pp. 423–431). New York: Liss.
- Shaw, R. E., McIntyre, M., and Mace, W. (1974). The role of symmetry in event perception. In R. MacLeod and H. Pick (Eds.), *Studied in perception: essays in honor of J. J. Gibson*. New York: Cornell University Press.
- Shaw, R. E., and Pittenger, J. B. (1977). Perceiving the face of change in changing faces: implications for a theory of object perception. In R. E. Shaw and J. Bransford (Eds.), *Perceiving, acting and knowing: toward an ecological psychology*. Hillsdale, NJ: Erlbaum.
- Shaw, R. E., and Pittenger, J. B. (1978). Perceiving change. In H. L. Pick and E. Saltzman (Eds.), *Modes of perceiving and processing information* (pp. 187–204). New York: Wiley.
- Shaw, R. E., Turvey, M. T., and Mace, W. M. (1981). Ecological psychology: the consequence of a commitment to realism. In W. Weimer and D. Palermo (Eds.), *Cognition and the symbolic processes II*. Hillsdale, NJ: Erlbaum.
- Solomon, H. Y. (1988). Movement-produced invariants in haptic explorations: an example of a self-organizing, information-driven, intentional system. *Human Movement Science*, 7, 201–224.
- Szűcs, E. (1980). *Similitude and modelling*. Amsterdam: Elsevier.
- Taylor, E. F., and Wheeler, J. A. (1966). *Spacetime physics*. San Francisco: Freeman.
- Thompson, J. M. T., and Stewart, H. B. (1986). *Nonlinear dynamics and chaos: geometrical methods for engineers and scientists*. New York: Wiley.
- Todd, J. T. (1981). Visual information about moving objects. *Journal of Experimental Psychology: Human Perception and Performance*, 7, 795–810.
- Todd, J. T. (1982). Visual information about rigid and non-rigid motion: a geometric analysis. *Journal of Experimental Psychology: Human Perception and Performance*, 8, 238–252.
- Todd, J. T. (1983). Perception of gait. *Journal of Experimental Psychology: Human Perception and Performance*, 9, 31–42.
- Todd, J. T., and Akerstrom, R. A. (1987). Perception of three-dimensional form from pattern of optical texture. *Journal of Experimental Psychology: Human Perception and Performance*, 13, 242–255.
- Todd, J. T., Mark, L. S., Shaw, R. E., et al. (1980). The perception of human growth. *Scientific American*, 242, 106–114.
- Todd, J. T., and Reichel, F. D. (1989). Ordinal structure in the visual perception and cognition of smoothly curved surfaces. *Psychological Review*, 96, 643–657.
- Todd, J. T., and Warren, W. H. (1982). Visual perception of relative mass in dynamic events. *Perception*, 11, 325–335.
- Tufillaro, N. B., Abbott, T., and Reilly, J. (1992). *An experimental approach to nonlinear dynamics and chaos*. Redwood City, CA: Addison-Wesley.
- Turvey, M. T., Shaw, R. E., Mace, W. M., et al. (1981). Ecological laws of perceiving and acting: in reply to Fodor and Pylyshyn (1981). *Cognition*, 9, 237–304.
- Ullman, S. (1979). *The interpretation of visual motion*. Cambridge, MA: MIT Press.
- Ullman, S. (1984). Maximizing rigidity: the incremental recovery of 3-D structure from rigid and nonrigid motion. *Perception*, 13, 255–274.

mic Press.

g gr
(pp. 423-

ception. In
New York:

zing faces:
Perceiving,

man (Eds.),

onsequence
the symbolic

xample of a
201-224.

ical methods

Psychology:

ric analysis.

reception and

n pattern of
ce, 13, 242-

th. *Scientific*

nd cognition

amic events.

near dynamics

receiving and

re from rigid

Verri, A., and Poggio, T. (1987). Against quantitative optical flow. In *First International Conference on Computer Vision* (pp. 171-180). London: IEEE.

Verri, A., and Poggio, T. (1989). Motion field and optical flow: qualitative properties. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 11*(5), 490-498.

Wallach, H., and O'Connell, D. N. (1953). The kinetic depth effect. *Journal of Experimental Psychology, 45*, 205-217.

Warren, R. (1990). Preliminary questions for the study of egomotion. In R. Warren and A. H. Wertheim (Eds.), *Perception and the control of self-motion* (pp. 3-32). Hillsdale, NJ: Erlbaum.

Warren, W. H., Kim, E. E., and Husney, R. (1987). The way the ball bounces: visual and auditory perception of elasticity and control of the bounce pass. *Perception, 16*, 309-336.

Warren, W. H., and Shaw, R. E. (1985). Event and encounters as units for analysis for ecological psychology. In W. H. Warren and R. E. Shaw (Eds.), *Persistence and change* (pp. 1-27). Hillsdale, NJ: Erlbaum.

Watson, J. S., Banks, M. S., von Hofsten, C., et al. (1992). Gravity as a monocular cue for perception of absolute distance and/or size. *Perception, 21*, 69-76.

Waxman, A. M., and Ullman, S. (1985). Surface structure and three-dimensional motion from image flow. *The International Journal of Robotics Research, 4*(3), 72-94.

Whittaker, E. T. A. (1944). *A treatise on the analytical dynamics of particles and rigid bodies*. New York: Dover.

Zacharias, G. L. (1990). An estimation/control model of egomotion. In R. Warren and A. H. Wertheim (Eds.), *Perception and the control of self-motion* (pp. 425-459). Hillsdale, NJ: Erlbaum.

Guide to Further Reading

A good general introduction to event perception is Warren and Shaw (1985). This anthology contains papers by researchers in the various sensory modalities and subspecialties (e.g., development, action, cognition, and language, as well as vision) and includes a review by Johansson. Michotte (1963) is the classic work on the perception of causality. An influential and very enjoyable introduction to scaling is Thompson (1961), while the most useful recent text is Szűcs (1980). Schroeder (1991) ranges widely over applications in various sciences, including psychology. For introductory works on ecological psychology, no books are more readable or more worth rereading than Gibson (1966, 1979). Reed (1988) traces Gibson's intellectual development and places his ideas in historical perspective. There are now many works available on nonlinear dynamics. Thompson and Stewart (1986) remains the most readable, yet fairly thorough introduction. Rosenberg (1977) is a good presentation of classical mechanics with a treatment of the various spaces in which events might be represented. Marmo, Saletan, Simoni, et al. (1985) provide the best overview of the strictly qualitative approach to dynamics. This work is rather technically demanding, but still readable. A useful general introduction to the mathematics of form is Lord and Wilson (1986). For a slightly more technical introduction to optical flow than Gibson (1979) or Reed (1988), see Nalwa (1993). For the full dose, see Horn (1986). Norman and Todd (1992) is also a good brief introduction to more advanced topics. Cutting (1986) provides a useful introduction to optics.

Cutting, J. E. (1986). *Perception with an eye for motion*. Cambridge, MA: MIT Press.

Gibson, J. J. (1966). *The senses considered as perceptual systems*. Boston: Houghton Mifflin.

Gibson, J. J. (1979). *The ecological approach to visual perception*. Hillsdale, N.J.: Erlbaum.

Horn, B. K. P. (1986). *Robot vision*. Cambridge, MA: MIT Press.

- Lord, E. A., and Wilson, C. B. (1986). *The mathematical description of shape and form*. New York: Wiley.
- Marmo, G., Saletan, E. J., Simoni, A., et al. (1985). *Dynamical systems: a differential geometric approach to symmetry and reduction*. New York: Wiley.
- Michotte, A. (1963). *The perception of causality*. London: Methuen.
- Nalwa, V. S. (1993). *A guided tour of computer vision*. Reading, MA: Addison-Wesley.
- Norman, J. F., and Todd, J. T. (1992). The visual perception of 3-dimensional form. In G. A. Carpenter and S. Grossberg (Eds.), *Neural networks for vision and image processing*. Cambridge, MA: MIT Press.
- Reed, E. S. (1988). *James J. Gibson and the psychology of perception*. New Haven, CT: Yale University Press.
- Rosenberg, R. M. (1977). *Analytical dynamics of discrete systems*. New York: Plenum.
- Schroeder, M. (1991). *Fractals, chaos, power laws*. New York: Freeman.
- Szűcs, E. (1980). *Similitude and modelling*. Amsterdam: Elsevier.
- Thompson, D. W. (1961). *On growth and form*. Cambridge, England: Cambridge University Press.
- Thompson, J. M. T., and Stewart, H. B. (1986). *Nonlinear dynamics and chaos: geometrical methods for engineers and scientists*. New York: Wiley.
- Warren, W. H., and Shaw, R. E. (1985). *Persistence and change*. Hillsdale, NJ: Erlbaum.

Mind as Motion

Explorations in the Dynamics of Cognition

edited by Robert F. Port and Timothy van Gelder

1995

A Bradford Book
The MIT Press
Cambridge, Massachusetts
London, England

he
to
nd
on-
om
ew