

Monocular Egocentric Distance Information Generated by Head Movement

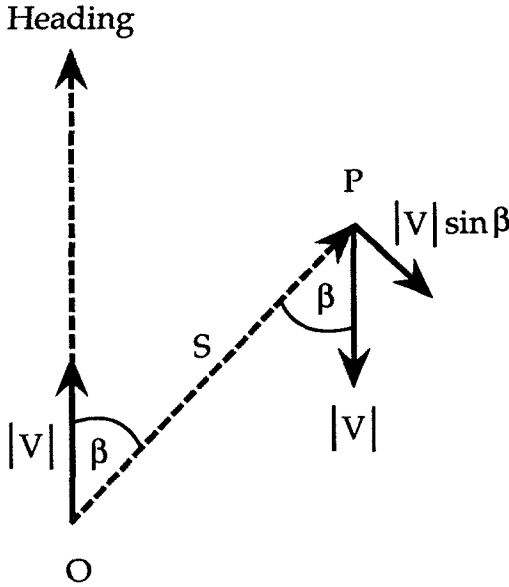
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How might definite spatial scale be determined on the basis of monocular optic flows? The evidence for the perception of distances in depth from optic flow indicates that head motion is essential for accurate performance. Accordingly, we propose a solution that entails the use of symmetries in the physically determined form of oscillatory head trajectories. The hypothesized solution is notable in two other respects. The first is the use of a time-dimensioned optic variable, τ (Lee, 1974). This strategy takes advantage of the fact that metric temporal scaling is preserved in optic flow. Second, somatosensory information about the amplitude of head motion is used to scale distance in intrinsic units. An alternative multimodal solution would require somatosensory information about velocity of head motion, but the evidence from studies of vestibulo-ocular reflex indicate that adequate velocity information may not be available. Finally, we discuss means for testing the proposed solution.

The essential problem in obtaining information about definite, metric distances from optic flow has been described in various elaborated treatments of optic flow (Koenderink, 1986; Koenderink & van Doorn, 1975; Simpson, 1993; Waxman & Ullman, 1985), but was most simply and elegantly described by Nakayama and Loomis (1974; see also Whiteside & Samuel, 1970). As shown in Figure 1, the velocity of relative motion between an observer and a point in the surroundings is projected into optic flow by dividing by the distance between the observer and the point. Dimensionally, the length unit associated with distance, S , and velocity, V , cancels so that optic velocity is in reciprocal time units (i.e., per time).¹ The spatial metric is lost in the mapping to optic flow (although a temporal metric is preserved).

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¹Angles are treated as dimensionless quantities.



$$\omega = \frac{d\beta}{dt} = \frac{|V| \sin \beta}{S}$$

$$S_i = \frac{|V| \sin \beta_i}{\omega_i} \quad i = 1, 2$$

$$\frac{S_1}{S_2} = \frac{\omega_2 \sin \beta_1}{\omega_1 \sin \beta_2}$$

FIGURE 1 The geometry of optic flow from observer movement. The observer O moves at velocity V past a point P in the surround that is at a distance S and at angle β to the direction of heading. The optic velocity of the point projected from P is $d\beta/dt = \omega$. The distance S would be specified in terms of optic position ($\sin\beta$) and optic velocity (ω) if V were known. Without V, the relative distance (S_1/S_2) of two points is specified by the corresponding optic positions and velocities (after Nakayama & Loomis, 1974).

According to this analysis, spatial quantities are specified in optic flow only to within a scale factor. Hypothetically, supplied with such a scale factor, one could recover definite spatial magnitudes. As shown in Figure 1, the ratio of two quantities, the optic position and the optic velocity, would specify the egocentric distance to an environmental point if the momentary velocity of the observer was known. The velocity would act as a scaler introducing a spatial metric (and associated unit). So, velocity given in meters per second would yield distance to an environmental point in meters. The difficulty is that no adequate source of information about the momentary metric velocity of the observer is known.

THE EVIDENCE

For the purposes of this article, we restrict consideration to the case where a monocular observer moves through an otherwise unmoving surround. Structure in optic flow that is specific to the structure of the surroundings is generated by translation of the point of observation. The ability to estimate distances in depth has been studied extensively in circumstances where the observer is translating laterally with respect to two or more points distributed at different distances from the observer. This generates *motion parallax*. Using the geometry in Figure 1, this occurs with point P near $\beta = 90^\circ$, so that $\sin\beta \approx 1$ and $d\beta/dt = \omega \approx V/S$. The corresponding optic velocities decrease with increasing distance of point P . Points distributed along a ground plane, for instance, yield a gradient of optic flow velocities akin to a ground texture gradient. Like the ground texture gradient, the flow gradient by itself can only provide information about relative distances in depth. The problem is to obtain information about definite distances.

Distance Perception

To date, studies have revealed various abilities to judge distance in depth from motion parallax depending on the display and the response measure. First, displays have been generated using either parallel or polar projection. *Parallel projection* approximates viewing of objects at relatively large distances so that images subtend only small visual angles. *Polar projection* is strictly correct at all distances and is necessary to represent viewing at closer distances. Motion parallax under parallel projection provides information about separation in depth, but the direction of depth order is ambiguous. So, in this case, reversals in depth order are common (e.g., Norman & Todd, 1992; B. Rogers & Graham, 1979; S. Rogers & B. Rogers, 1992; Todd & Bressan, 1990; Todd & Norman, 1991; Todd & Reichel, 1989). Second, optic flow has been generated by and coupled to head movement in some displays and not in others. For instance, motion parallax under parallel projection coupled to head motion has been found to yield stable depth order (S. Rogers & B. Rogers, 1992). Third, two different types of distance have been investigated. The first is *egocentric* distance from the observer to a point in the environment. The second is *exocentric* distance between two environmental points that are separated in depth from one another. Johnston (1991) showed that the two types of distance are related. In a study on stereopsis, she found that perceived surface curvature in depth (effectively, exocentric distance) was predicted by results from studies on stereoptic perception of egocentric distance. Fourth, some studies of egocentric distance perception have used a single light point in a dark field; others have used more than one point or extended optic structure. Finally, observers have

expressed perceived distance either verbally or by various nonverbal means. Verbal estimates have exhibited greater constant and random error.

The majority of the extant studies of depth perception via motion parallax have used judgments of exocentric distances. Many such studies of the *kinetic depth effect* (KDE), or *structure-form-motion*, have used parallel projection without head movement. These are less useful for our purposes. Studies of exocentric distance under polar projection without head movement have produced good judgments of depth order and relative distance at small distances, but at larger distances both relative and definite, metric distances have been underestimated by as much as 50% to 80% (Braunstein & Tittle, 1988; Eby, 1992; Lappin & Fuqua, 1983; Lappin & Love, 1992; Ono & Steinbach, 1990; B. Rogers & Graham, 1979; S. Rogers & B. Rogers, 1992). Polar projection with head movement has generally produced the most accurate estimates of depth order or of definite exocentric distances (Ono & Steinbach, 1990; B. Rogers & Graham, 1979; S. Rogers & B. Rogers, 1992). Nevertheless, Steinbach and Ono (1991) found that, despite accurate estimates for smaller distances (4 to 8 cm), larger exocentric distances (20 cm) were underestimated by half.

Studies of egocentric distance perception are fewer in number and all use polar projection. Gogel and Tietz (1973, 1979) found that verbal estimates of the distance of a single light point viewed in a dark surround with head movement are highly variable, underestimate distances beyond 0.8 to 2 m, and overestimate distances inside this distance. This pattern is consistent with results of Eriksson (1974), Ferris (1972), Foley (1977, 1978), and Foley and Held (1972).² With observer head motion along the depth axis, Eriksson (1974) found that verbal estimates of targets at 3 to 7 m underestimated distances with a slope in the relation between actual and estimated distance of .7. Ferris (1972) performed studies in which observers made verbal judgments of egocentric distances both with and without head movement. Without head movements, judgments were extremely poor. With head movement and without training, estimates were accurate for near distances and exhibited progressively greater underestimation at increasing distances. The slopes of judgment curves were from .4 to .7. With training, however, slopes increased to between .8 and 1, so that the tendency for progressive underestimation was eliminated.

²Foley (1977) obtained this pattern of results when observers viewed targets (< 1 m) monocularly without head motion but with eye movement. He referred to this as "monocular parallax" because there was no evidence that accommodation would yield information about distance (Foley, 1978). Bingham (1993a) investigated the perception of depth via optic flow produced by eye movement, calling it "ocular parallax." Eye movement translates the effective point of observation in the entrance pupil because it is displaced from the center of rotation by about 11 mm. Under conditions tested by Bingham, observers could detect separation in depth but not depth order. Both Foley (1977) and Eriksson (1974), however, found that observers could judge depth order, producing a pattern of verbal estimates like Gogel's (Gogel & Tietz, 1973, 1979). When Gogel arranged displays so that "ocular parallax" was inconsistent with monocular parallax from head movement, the resulting judgments failed to vary with distance (Gogel & Tietz, 1979).

This latter result implies that the progressive underestimation found in many studies may be a reflection of the need to transform apprehended distances to extrinsic units (e.g., feet or meters) for expression in verbal judgments. The distance estimates may be more accurate when made using units intrinsic to the perception and action systems. As Lappin and Love (1992) noted, the poor performance exhibited in all these studies is inconsistent with performance in many common activities like stepping upward to climb a stair or reaching to grasp an object.³ Loomis, DaSilva, Fujita, and Fukusima (1992) replicated a progressive underestimation of distances in depth using explicit verbal judgments; but when they tested the apprehension of the same distances using an action measure, the errors were eliminated. Foley (1977, 1985), in studies of binocular distance perception, also found differences in the pattern of data from manually indicated distances as opposed to verbal judgments. For instance, variability was lower in the manually expressed estimates and did not increase with distance as did variability of verbal estimates. Although the slopes of manual judgment curves were less than 1, they were twice as steep as verbal judgment curves. Gogel and Tietz (1979) obtained accurate nonverbal estimates of egocentric distances from 30 cm to 100 cm using a single light point in a dark surround with head movement. Johansson (1973) also obtained very accurate nonverbal estimates of egocentric distances up to 2 m using a rectangular array of four points and head movements of about 1 cm in amplitude. Without the head movement, estimates did not vary with actual distances. (In both of the latter studies, binocular viewing reduced random error with equally small constant error.)

Action response measures may better indicate the ability to apprehend distances in depth by eliminating the need for an accessory, ill-constrained transformation to extrinsic units. For instance, the scaling of distance in terms of control parameters for reaching would be highly nonlinear with overlapping discontinuous regions determined by the articulators used to reach a distance (e.g., arm only vs. arm and trunk vs. arm, trunk, and legs). Also the intrinsic scaling of reach space must reflect structure determined by the dynamics of the arm, its stiffness, damping, and inertial characteristics, all of which vary

³For two reasons, good performance in these cases cannot be attributed simply to the use of binocular vision. First, a substantial proportion of the general population has no binocular vision. Including anisometropes (who have lost stereopsis to asymmetric refractive errors in the two eyes), amblyopes (who have lost one eye to neurological dysfunction), and strabismics (who fail to converge their eyes properly due to muscular problems), as well as those who have outright lost an eye, a conservative estimate is that between 10% to 20% of the population is monocular (Borish, 1970). These individuals perform myriad tasks requiring good apprehension of definite distances in depth nevertheless. Second, the precision of stereopsis is variable especially away from the horopter. Stereopsis is easily overpowered by other sources of information, especially optic flows. For instance, Lappin and Love (1992) found stereopsis to be completely dominated when in competition with monocular optic flow and ineffective as a substitute for the monocular information when it was removed.

nonlinearly through the space (Hogan, 1985, 1990; Hogan, Bizzi, Mussa-Ivaldi, & Flash, 1987; Hogan & Winters, 1990; Kay, Hogan, Mussa-Ivaldi, & Fasse, 1989a, 1989b; Mussa-Ivaldi, Hogan, & Bizzi, 1985). Judgments in units of arm length may be somewhat more intuitive, but cannot be isomorphic to an intrinsic action scale which has, for instance, absolute maximum as well as minimum values (Bingham, 1993c). An arm-length scale would be isomorphic to extrinsic scales which have no maximum values (e.g., $2 \times$ arm length ≈ 0.8 m and $200 \times$ arm length ≈ 80 m). Both the systematic and random errors typical of verbal judgments may reflect the nonlinear relation between the scale used to express judgments verbally and the scale intrinsic to the control of relevant actions.

Vestibular System and Kinesthesia

Optic flows generated by head movement have been shown to allow potentially good apprehension of distance. However, we have no theoretical understanding of how this might be achieved. We know that motion parallax might be scaled to yield definite, metric distances if the momentary velocity of the point of observation were knowable, but we do not know how the velocity could be known. Studies of the vestibulo-ocular reflex (VOR) indicate that some information about velocity of head rotation is available. The VOR helps one to maintain fixation on a point in the surround by counterrolling the eyes as the head is rotated. Single-cell recordings of neurons projecting from the semicircular canals in mammals reveal activity that is proportional to angular head velocity (Reisine & Highstein, 1981). Because the semicircular canals are known to act as angular accelerometers (Benson, 1990), this result implies that the output is integrated to yield angular velocity. The problem is that although the recordings are more or less proportional to angular velocity (there are regular distortions in the signal in addition to noise), the accuracy and stability of the scaling to actual velocity remains at issue (i.e., a proportionality constant or, alternatively, a constant of integration must be determined). The evidence for such scaling must be sought in studies of the VOR. The problem is here complicated by the fact that the eyes are displaced from the vertical axis of rotation in the head by about 10 cm. This means that the eyes must counterroll through a greater angle than that of head rotation to maintain fixation (Berthoz, 1985). Furthermore (and here is the crux), the required angle of eye rotation depends on the distance of the fixated point.

Counterrolling of the eyes to maintain fixation on an imagined target in the dark yields a gain (i.e., relating head to eye rotation) of about .90 with active head rotation (Jell, Guedry, & Hixson, 1982) and of only .75 with passive head rotation (Benson, 1970; Barnes, 1979). Although the gain with an actual fixation target is close to 1, both the magnitude and the sign of the gain can be altered when targets are viewed through prisms or reversing lenses (Berthoz,

1985). Most important, Biguer and Prablanc (1981) showed that the amplitude of eye rotation (or the gain) varied depending on the distance of a fixation target and that the variation was dependent on the perception of the target distance rather than on retinal slip. This means that information from the semicircular canals is scaled to produce appropriate counterrolling of the eyes using visual information about target distance. The implication is that information about distance may be used to scale information about head velocity rather than the reverse.

The analysis thus far applies only to rotation of the head. Head translation entails the use of information from the saccule and utricle, the otolith organs. These are polarized linear accelerometers which respond in proportion to linear acceleration from translational movement in a given direction (or rotational movement) and from gravity (Benson, 1990; Miles, 1993). Eye fixation in the case of head translation is even more complicated. It depends on both the distance and the direction of the fixation target with respect to the direction of head translation. For a point on the ground to the left of the direction of heading, the eye must rotate downward to the left. For a point on the ceiling to the right, the eye must rotate upward to the right and so on. In all cases, the amount of rotation depends strongly on distance of the fixation target and numerous studies have found scaling proportional to distance (Paige, 1988; Paige & Tomko, 1991; Schwarz, Busetini, & Miles, 1989; Schwarz & Miles, 1991). The otolith organs may contribute to this response at higher frequencies of head motion although the effect of gravity on the otoliths will still be part of their response pattern. The otoliths have been found to contribute primarily to postural maintenance and, accordingly, the otoliths contribute to ocular compensation for head tilt at lower frequencies of head motion (Miles, 1993). The most obvious difficulty is that vestibular organs are accelerometers and, therefore, respond only to changes in velocity. During locomotion, for instance, output is only proportional to changes about the mean velocity of translation and no information about the mean velocity itself is provided. The availability of accurate information about translational head velocity from the vestibular system is doubtful and, in any case, largely unknown. In addition, there is no evidence that accurate velocity information might be available via joint, cutaneous, or muscle kinesthesia.

In contrast, the ability to determine amplitude of motion via kinesthesia is well established (Clark & Horch, 1986; McCloskey, 1980). Furthermore, voluntary movement to an endpoint at preferred rates of motion has been found to be optimal for yielding accurate judgments of position via kinesthesia (Clark & Horch, 1986; Eklund, 1972; Paillard & Brouchon, 1968, 1974). Recall that the origin of the distance problem is in the loss of spatial (or length related) metrics in the mapping from surrounding surfaces into optic pattern. Intrinsically scaled length can be obtained via somatosensory kinesthesia of head movement. The problem, then, is to find a way to use amplitude of head movement to scale optic

information about egocentric distance. We accomplish this by using an otherwise much studied optic variable, namely τ .

Sensitivity to τ and Its Use

Although spatial metrics are lost in the mapping to optic flow, we have noted that temporal metrics are preserved. Therefore, a logical possibility for scaling behavior appropriately to contact with surrounding surfaces is to use time-dimensioned optic variables. This was the strategy adopted by Lee (1974, 1980) who analyzed the case where an observer is translating directly toward points in the surround. In Figure 1, this corresponds to β small so that $\sin\beta \approx \beta$ and $S/V = \beta/\omega = \tau$. τ is a time-dimensioned optic variable that corresponds (when V is constant) to the time-to-contact between observer and a surface in the surround. Todd (1981) showed that human observers can reliably detect and use τ differences of about 150 ms. Further, Regan and Hamstra (1993) showed that τ is detected with a resolution described by a Weber constant of about 10%. Other studies have demonstrated the use of τ in a wide variety of visually guided actions (Bootsma & Peper, 1992; Bootsma & van Wieringen, 1990; Lee, 1980; Savelsbergh, Whiting, & Bootsma, 1991; Sidaway, McNitt-Gray, & Davis, 1989; Todd, 1981).

Using Movement Toward a Target Rather Than Lateral Movement

Of those studies on distance perception that we described as using head movement, all except one (viz., Eriksson, 1974) employed lateral motion of the head with respect to the direction of the target. In contrast, τ entails movement directly toward a target. Using optic flows generated by translation toward a surface makes sense in the case of targeted actions because such motion also provides reliable information about the direction to a target. The *visual direction* is the direction from which light is projected through the point of observation in the entrance pupil of the eye. Although, the visual direction provides a source of information about direction that is available in static images (Howard, 1982), a large number of experiments performed using displacement prisms have shown that observers can perceive the actual direction to a target despite perturbation of the visual direction (Howard, 1982; Welch, 1978). Bingham and Romack (1992) showed that practiced observers can adjust immediately to perturbations of visual direction. With head movement toward a target, the optic pattern exhibits a radial outflow from a node that lies in the target image. Warren and colleagues have shown that observers can use this information to judge the direction of heading within about 1° accuracy (Warren & Hannon, 1990; Warren, Mestre, Blackwell, & Morris, 1991; Warren, Morris, & Kalish, 1988). The nodal point in the optic flow only lies in the target image when the head is in fact moving toward the target even if the target image has been displaced by a prism.

Tharp, Liu, and Stark (1992) performed experiments employing virtual reality displays in which head movements were not restricted or prescribed. They reported that participants used head movements directed predominantly along a line to a target. We have also performed pilot studies in which head motion was required to reveal target distances and in which participants reported that head motion toward the target was most useful (Bingham, 1993b).

Targeted Reaching, Head Movement, and Neck Kinesthesia

Targeted reaching does indeed require information about both target distance and direction. Although vision of the target during a reach is not required to bring the hand within the ballpark of a target (Goodale, Pelisson, & Prablanc, 1986; Pelisson, Prablanc, Goodale, & Jeannerod, 1986), performance with vision of the target is considerably more accurate especially with free head movement (Biguer, Prablanc, & Jeannerod, 1984; Carlton, 1992; Carnahan, 1992; Carson, Goodman, Roneo, & Elliot, 1993; Elliot & Allard, 1985; Jeannerod & Prablanc, 1983; Prablanc, Echalié, Jeannerod, & Komilis, 1979; Prablanc, Echalié, Komilis, & Jeannerod, 1979; Proteau & Cournoyer, 1990; Sivak & Mackenzie, 1992). The control of head posture and movement is an important part of reaching (Marteniuk, 1978). Biguer, Donaldson, Hein, and Jeannerod (1986) vibrated the muscles (and spindle afferents) in the back of the neck during reaching to produce an illusory perception that the neck muscles were extending. The result was a consistent overshoot in reaching.⁴

Egocentric Distance From Head Movement Toward a Target at Preferred Rates

Head movement toward a target provides at least two types of information: (a) information about the direction of the target from the observer and (b) information about the time-to-contact with the target. As we next show, however, head movement toward a target can yield specification of the distance to the target as well. If we treat head motion as generated by a simple physical oscillator, the symmetries of the oscillatory trajectories can be used to derive a relation that specifies the distance to a target in units intrinsic to the head movement (Bingham, 1993d).

When the head oscillates toward and away from an object, τ at the peak velocity (τ_{pv}) of head movement is midway in time or in position between the

⁴Head movement has been found to be required for accurate performance in other scaling tasks as well. For instance, Mark, Balliett, Craver, Douglas, and Fox (1990) investigated judgments of maximum seat height that one could sit on without climbing and found that without free head movements judgments were unstable and inaccurate.

endpoints of the oscillatory motion and, thus, can be easily found and used. The value of τ_{pv} divided by the period of oscillation (T) is equivalent to the distance of the object in units of the amplitude of head oscillation (scaled by 2π). Thus, with oscillation of the head toward and away from an object, $2\pi\tau_{pv}/T$ specifies that the object is twice as far as one is moving one's head, or three times as far, and so on. τ_{pv} is optically specified.

The amplitude of head motion (as well as the midpoint of motion) might be determined via muscle, cutaneous, and joint afferents. The locus of the peak velocity of motion might also be determined via the vestibular system as the point of transition from positive to negative acceleration in the direction of motion. Alternatively, the amplitude of head motion could be specified optically in terms of the extent of occlusion of the shoulder and arm at the edge of the visual field, that is, in units relevant to reaching (i.e., percentage of arm length).

Determination of the period of motion could entail either a source of temporal metric intrinsic to the control of head movement (e.g., stiffness) or the use of a biological clock. The biological clock could, in principle, be a strictly neural entity (i.e., the internal clock of motor programming theory). Alternatively, rhythmic head motion might provide its own clocking metric. Studies of rhythmic limb movements have shown that periods of motion at preferred rates are extremely stable and produced with great reliability by a given individual on occasions separated by as much as a year (Bingham, Schmidt, Turvey, & Rosenblum, 1991; Kugler & Turvey, 1987). The implication is that preferred periods are recognizable. If so, then they might be used to index nonpreferred periods. Finally, if head oscillation were produced reliably at a single stable preferred period, then this aspect could be absorbed into a constant scaling coefficient that would be tuned with practice. The latter possibility would result in greater predicted random error in perceived distances.

Finally, although the analysis entails oscillatory movement, more than a single cycle of movement might only provide redundant information. In principle, a single half-cycle of movement toward a target could suffice.

MATHEMATICAL DERIVATION

As shown in Figure 2, we assume the head to be moving along a line of sight directed to a target surface and model the movement as generated by a harmonic

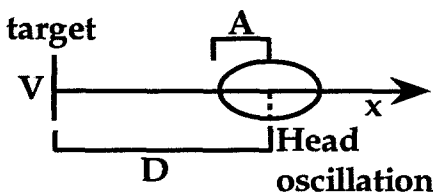


FIGURE 2 Hypothetical phase portrait of head oscillation with an equilibrium point at distance D from a visual target. Variation in velocity of head motion follows an elliptical trajectory with an amplitude A along the x position coordinate.

mass-spring dynamic.⁵ In Figure 2, the head moves with an amplitude, A , about the equilibrium point of the mass-spring which is located at a distance D from the target. A phase-space trajectory is shown with variation in position, x , along the abscissa and variation in velocity, V , along the ordinate. The target is located at $x = 0$.

The dynamic equation of motion for this mass-spring is

$$\ddot{x}(t) = -\frac{k}{m}[x(t) - D] \quad (1)$$

where k is a linear stiffness and m is the mass, both assumed constant. Using ω for the frequency and T for the period, with

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} \quad (2)$$

we integrate Equation 1 to derive equations for the velocity and position:

$$\dot{x}(t) = -A\omega \cos(\omega t) \quad (3)$$

$$x(t) = D - A\sin(\omega t). \quad (4)$$

Peak velocity is reached at the midpoint of the half-cycle, where $\cos(\omega t) = 1$ and $\sin(\omega t) = 0$, so assuming motion toward the target, we obtain

$$\dot{x} = A\omega = A \frac{2\pi}{T} \text{ and } x = D. \quad (5)$$

Given $\tau = x/\dot{x}$, we substitute in this relation using Equation 5 to obtain

$$\tau_{pv} = \frac{D}{A \frac{2\pi}{T}} \quad (6)$$

where τ_{pv} is τ at peak velocity of the oscillation. Dividing through by T and rearranging yields

⁵For a mass-spring model of head movement, see Bizzi, Polit, and Morasso (1976). Trajectories modeled for our purposes here as a harmonic or linear, undamped mass-spring can also be treated as an approximation to limit cycle trajectories generated by more complex nonlinear dynamics used to model movements of other limbs (see, e.g., Beek, 1989; Beek, Turvey, & Schmidt, 1992; Kay, Kelso, Saltzman, & Schönner, 1987; Kay, Saltzman, & Kelso, 1991).

$$\frac{\tau_{pv}}{T} = \frac{1}{2\pi} \frac{D}{A}. \quad (7)$$

That is, τ at the peak velocity of oscillation divided by the period of motion is equivalent to the distance to the target in head amplitude units scaled by $1/2\pi$.

SIMULATION AND ILLUSTRATION

We performed simulations to illustrate the invariance described in Equation 7. Using 3 stiffness values, 2 amplitudes of motion, and 4 distances from the target, we produced the half-cycle of oscillatory motion toward the target for each configuration of values. In Figure 3, the trajectories appear against corresponding distances from the target. We then divided $x(t)$ by $v(t)$ along each trajectory to produce the τ trajectories shown in Figure 4. It is interesting to note in this figure that by extrapolating along successive positions of an oscillator of constant stiffness and amplitude, one can follow a time-to-contact trajectory

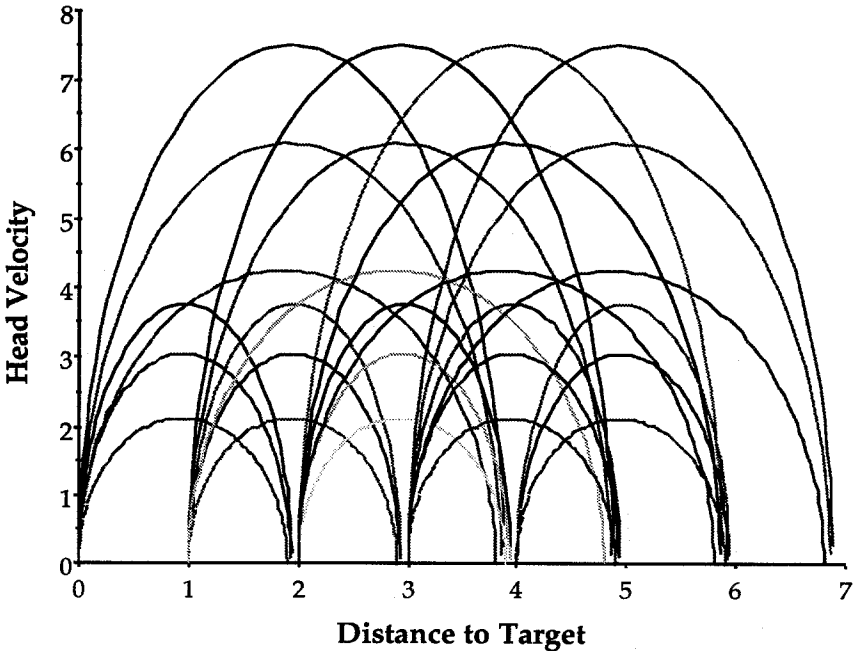


FIGURE 3 Phase portraits for half-cycles of simulated head oscillation directly toward a target at 4 distances from the target and with 2 different amplitudes and 3 periods of motion.

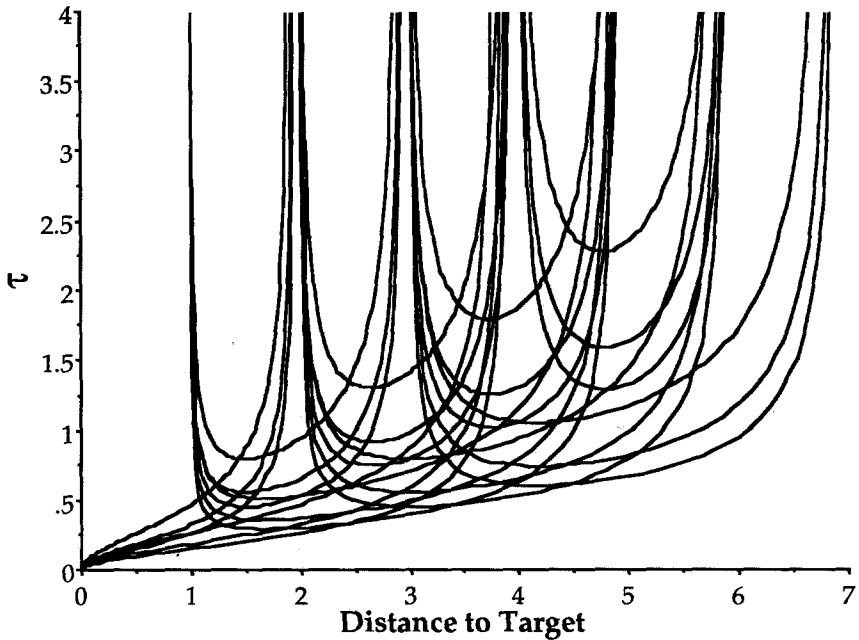


FIGURE 4 τ trajectories plotted for the same motions as simulated in Figure 3. τ computed as the momentary distance to the target, x , divided by the velocity toward the target, dx/dt .

into the target. On the other hand, as the stiffness and amplitude of head motion are varied, the τ trajectories populate an indefinite and continuous variety of locations in the space. That is, there is no invariance obvious in this figure without a guarantee of stability and reliability of head motion of given stiffness and amplitude. This is also shown in Figure 5 where the τ_{pv} have been plotted against corresponding distances for all the trajectories.

Finally, however, when τ_{pv}/T is plotted against D/A in Figure 6, we see all the variability collapse, producing the predicted invariance across trajectories. A simple linear regression reveals the scaling constant equal to $1/2\pi$. The especial importance of the invariance illustrated in this figure is that the stiffness (e.g., frequency) and amplitude of the oscillator need not be stable over successive cycles or instances of movement as long as one continues to have information about the current period and amplitude. The relation follows the regression line for all values of stiffness and amplitude. This invariance is important in view of the fact that oscillatory limb motions exhibit a limit cycle stability with a stochastic component that results in the wandering of trajectories within a band in which both amplitudes and peak velocities vary from cycle to cycle (Kay, 1988; Kay, Saltzman, & Kelso, 1991). However, peak velocities

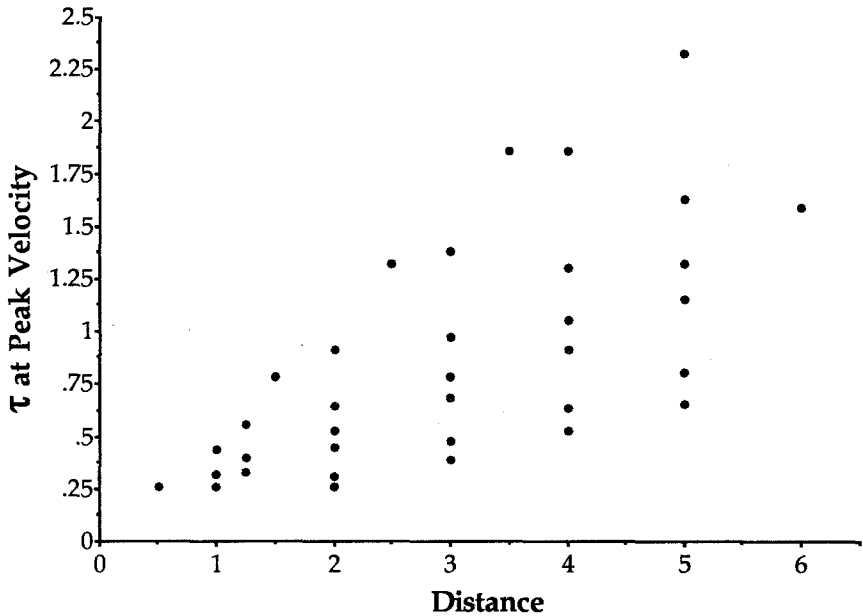


FIGURE 5 τ values taken from Figure 4 at a point along each trajectory corresponding to the peak velocities as they appear in Figure 3. τ_{pv} plotted against distance to the target.

and amplitudes tend to covary so as to produce a stable frequency of oscillation. The hypothesized information about distance would be especially stable given stability in frequency as long as information about the current amplitude is available.

REPRISE OF THE PROBLEM AND OUR METHOD OF SOLUTION

Another way to understand this solution is as follows. Noting once again that $\tau = x/\dot{x}$, with simple rearrangement we find that $x = \dot{x}\tau$. That is, just as has been shown in analyses of motion parallax (Koenderink, 1986; Nakayama & Loomis, 1974), if the momentary velocity of motion were known, then (using τ in this case) the distance from a target could be obtained. The problem is knowledge of the velocity. Velocity of head motion varies continuously in time and the ability to establish definite, metric velocity via the vestibular system or muscle, skin, and joint afferents with required accuracy is unknown. We solve this problem, in part, by taking advantage of the symmetries of the physical oscillator as reflected in Equation 5. τ at the peak velocity of oscillation (τ_{pv}) can be found easily as lying midway in both time and position between the endpoints of

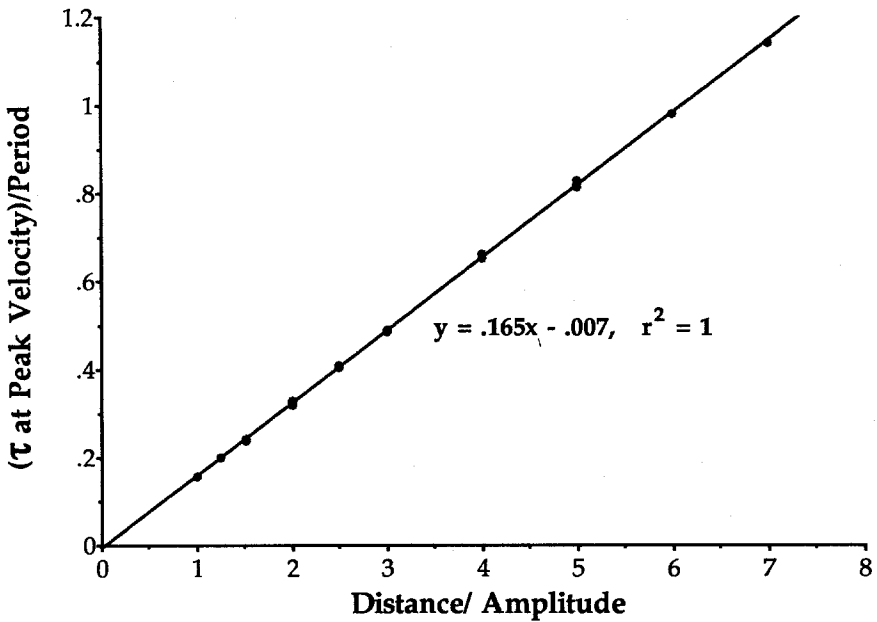


FIGURE 6 The τ values in Figure 5 divided by the corresponding periods of oscillation and plotted against the distance to the target divided by the amplitude of head oscillation. A line was fitted to the points using least-squares regression to reveal that the slope of the line is $1/2\pi$.

oscillation. The remaining key to this solution is to use a time-dimensioned variable whose scaling is preserved in optics and to scale distance in units intrinsic to the properties of the observer, that is, via kinesthetic apprehension of amplitude of head motion. The result is a dimensionless specification of distance or a π -number (Emori & Schuring, 1977; Warren, 1984) for egocentric distance. Alternatively, with specification of head movement amplitude in terms of the percentage of arm length progressively occluded at the edge of the field of view, distance can be specified in units of arm length.

Finally, this analysis could be applied to visually guided locomotion as well as to targeted reaching. In walking, for instance, the period of the head oscillation is determined by the simple equivalent pendulum length of the stance leg operating as an inverted pendulum. The amplitude of head motion would be equivalent to stride length, yielding a scaling of the surrounding terrain in units of stride length. With this last observation, we note that although the merging of perception and action is essential to our solution of the distance perception problem, we need not resort to an assumed knowledge of control parameter values to achieve our solution. Application to walking would require different parameters. Rather, we base our solution on the form of behavior itself, which

is necessary given its physical origin and which is something that the perceptual system could detect and use (Bingham, in press).

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