Another Timing Variable Composed of State Variables: Phase Perception and Phase Driven Oscillators

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ABSTRACT

In this chapter, we consider a perceptible variable that is related to τ, but is different from τ. The variable is phase, ϕ. ϕ is similar to τ in that both are timing variables and both are ratios of spatial variables that could be state variables of a dynamical system. As such, either could be used to drive a damped mass-spring system to yield an autonomous dynamical organization. Finally, both τ and ϕ are perceptible variables. We describe experiments in which we have investigated the perception of relative phase. Then, we describe a phase driven and phase coupled dynamical model of bimanual coordination. An important feature of this model is that it can account for both movement study and judgment study results. However, the way the perceptible property is used in each case is task-specific.

τ as a Temporal Variable Composed of Spatial State Variables

In perception / action research, there have been two especially salient reasons to hypothesize and investigate τ as a perceptible variable. Both reasons relate to the problem of timing actions and coordinating them with respect to objects in the surroundings that are moving relative to the performer. The first reason is that a temporal variable is most appropriate for the control of timing. Temporal variables are more commonly studied in audition than in vision where the prevailing focus has been on spatial variables. The problem in 'space perception' is that the length dimension is lost in optical structure which is intrinsically angular and temporal. (For extended discussion of the following, see Bingham (1988; and 1995).) Optical extents can be described in radians or degrees, but not centimeters or inches. Extra-optical variables must be considered when investigating visual perception of length related properties like size, distance, or velocity. Perhaps related to this fact is the recurrent finding that space perception is rather inaccurate or imprecise when apprehension of lengths (as opposed to length ratios) is required (e.g. Bingham & Pagano, 1998; Bingham, Zaal, Robin & Shull, 2000; Tittle, Todd, Perotti & Norman, 1995; Todd, Tittle & Norman, 1995). As an optical variable, τ can only be angular and temporal, but it is equivalent to the ratio of the distance and velocity of a surface moving towards an observer. In the ratio, the length dimension cancels leaving only time. If τ is used by performers to scale their actions to the surroundings, then the measurement problem in space perception can be avoided.

The second reason to study τ emerged as perception / action research began to focus on the issue of stability and the need to integrate perceptible variables into the underlying dynamics of action. The progenitor of task dynamic approaches to perception / action was the λ-model (or Equilibrium Point (EP) model) of limb movement (Feldman, 1980; 1986; Feldman, Adamovich, Ostry & Flanagan, 1990; Latash, 1993). Feldman showed that the muscles and the peripheral nervous system were organized to control joint motion as an abstract mass-spring organization parametrized by stiffness and the EP. Ignoring the differences among competing mass-spring models (e.g. α-model versus λ-model), its now clear that the damped mass-spring is fundamental to the organization of action (Bizzi, Hogen, Mussa-Ivaldi & Giszter, 1992;

Feldman, Adamovich, Ostry & Flanagan, 1990; Hogan, Bizzi, Mussa-Ivaldi & Flash, 1987; Latash, 1993). The advantages of this organization all amount to stability. The relatively autonomous peripheral organization entails both short neural transmission distances and use of intrinsic spring-like muscle properties to yield a nearly linear spring at the joint level (Latash, 1993). The organization combines postural control with the control of movement to yield fixed point stability or equifinality for end postures in discrete movements. Mass-spring organization yields stable posture, but the problem of movement stability remains.

As fixed point stability is desirable for posture, limit cycle stability is desirable for movements. In perturbation experiments, Kay, Saltzman and Kelso (1991) found that rhythmic limb movements exhibit limit cycle stability. If the mass-spring organization is to be used to account for rhythmic movement, then its necessary to drive the mass-spring to yield limit cycle stability (Schöner, 1990; Zaal, Bootsma & van Wieringen, 1999). The presence of limit cycle stability implies, in turn, that the dynamic is nonlinear (Jordan & Smith, 1977), that is, products or quotients of state variables appear in the dynamical equations. Kay, et al. (1991) also found that rhythmic limb movements exhibit phase resetting. This means that the mass-spring is driven in a way that preserves autonomous organization. The driver is not itself external to the oscillator dynamic, but instead must be a function of the behavior of the oscillator itself. The driver can not be a function of time, but instead must be a function of the state variables of the oscillator, namely, position and velocity (x[t], v[t]). Especially in tasks that require interaction or coordination with events in the surroundings (e.g. catching a ball), the dynamic must be driven perceptually. This can be accomplished using τ, because τ combines position and velocity in a quotient to yield time. This solution was investigated initially by Schöner (1991) and subsequently by others (Bingham, 1995; Zaal, Bootsma & van Wieringen, 1998). (See the chapters by F.T.J.M. Zaal and by G.P. Bingham and F.T.J.M. Zaal in the current volume.) So, the advantages of τ are that it is temporal (not spatial), but is nevertheless composed of spatial variables that can be the state variables in a dynamical (e.g. mass-spring) system.

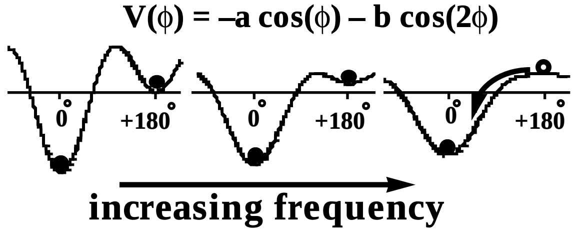
In the following, we introduce another variable that is, like τ, composed of a ratio of position and velocity, spatial variables that can be state variables in a dynamical system. Also like τ, therefore, this other variable can be used to drive a mass-spring system to yield limit cycle behaviors. The other variable is phase, ϕ. Unlike τ, however, ϕ has not been treated as a perceptible variable until fairly recently.

Relative phase and the HKB Model

In coordination of rhythmic bimanual movements, relative phase is the relative position of two oscillating limbs within an oscillatory cycle. For people without special skills (e.g. jazz drumming), only two relative phases can be stably produced in free voluntary movement at preferred frequency (Kelso, 1995). They are at 0° and 180°. Other relative phases can be produced on average when people follow metronomes, but the movements exhibit large amounts of phase variability (Tuller & Kelso, 1989). They are unstable. Preferred frequency is near 1 Hz. As frequency is increased beyond preferred frequency, the phase variability increases strongly for movement at 180° relative phase, but not at 0° (Kelso, 1990). If people are given an instruction not to correct if switching occurs, then movement at 180° will switch to movement at 0° when frequency reaches about 3-4 Hz (Kelso, 1984; Kelso, Scholz & Schöner, 1986; Kelso, Schöner, Scholz & Haken,1987). With the switch, the level of phase variability drops. There is no tendency to switch from 0° to 180° under any changes of frequency.

These phenomena have been captured in a dynamical model formulated by Haken, Kelso and Bunz (1985). The HKB model is a first order dynamic written in terms of the relative phase, ϕ, as the state variable.

FIGURE 1. The HKB model:
$V(\phi) = -a\cos(\phi) - b\cos(2\phi)$



The equation of motion, which describes the temporal rate of change in ϕ, that is, $\dot{\phi}$, is derived from a potential function, $V(\phi)$, which captures the two stable relative phases as attractors as shown in Figure 1. The attractors are wells or local minima in the potential layout. As the dynamic evolves, relative phase is attracted to the bottom of the wells at 0° and 180°. A noise term in the model causes the relative phase to depart stocastically from the bottom of a well. The effect of an increase in frequency is represented by changes in the potential. The well at 180° becomes progressively more shallow so that the stochastic variations in relative phase produce increasingly large departures in relative phase away from 180°. These departures eventually take the relative phase into the well around 0° at which point, the relative phase moves rapidly to 0° with small variation.

Investigating phase perception

The HKB describes the basic phenomena of bimanual coordination intuitively. People asked to oscillate their limbs stably at 0° or 180° know what to do, but if they are asked to oscillate at other phases, they do not. 0° and 180° are clearly delineated by the form of the potential function which represents the relative stability (or inversely, the effort) of oscillating at different relative phases. People seem to know where these phases are in a space of relative stabilities. Nevertheless, we wondered: what is the ultimate origin of the potential function in this model? Why are 0° and 180° the only stable modes and why is 180° less stable than 0° at higher frequencies? To answer these questions, we investigated the perception of relative phase because the bimanual movements are coupled perceptually, not mechanically (Kelso, 1984; 1995). The coupling is kinesthetic when the two limbs are those of a single person. Schmidt, Carello and Turvey (1990) found the same behaviors in a visual coupling of limb movements performed by two different people. Similar results were obtained by Wimmers, Beek, and van Wieringen (1992). To perform these tasks, people must be able to perceive relative phase, if for no other reason, than to comply with the instruction to oscillate at 0° or 180° relative phase.

Because the coupling is perceptual and because achievable phase relations seem to be specified in a space of relative stabilities (see Bingham, Zaal, Shull, and Collins (2001) for discussion), we investigated the visual perception of mean relative phase and of phase variability using both actual human movements (Bingham, Schmidt & Zaal, 1998) and simulations (Bingham & Collins, in preparation; Bingham, et al., 2001; Zaal, Bingham & Schmidt, 2000). Participants observed two disks oscillating on a computer monitor along straight horizontal paths, one above the other. (Zaal, et al. (2000) also investigated motions in depth which yield optical expansion and contraction, and replicated the results for motion in a frontoparallel plane.) In Zaal, et al. (2000) and Bingham, et al. (2001), the manipulated variables included mean relative phase (0°, 45°, 90°, 135°, 180°) and phase variability (0°, 5°, 10°, 15° phase SD). Bingham, et al. (2001) also manipulated frequency (0.75 hz and 1.25 hz). However, we will describe in detail results of Bingham and Collins (in preparation) which replicated the previous results, but extended the manipulation of frequency to 1 hz, 2 hz, and 3 hz. Different groups of ten participants each judged either mean relative phase or phase variability on a ten point scale: for mean phase, 1 = 0° and 10 = 180°; for phase variability, 1 = 'not variable' and 10 = 'highly variable'. Participants received extensive instruction and demonstrations distinguishing mean phase and phase variability. They performed blocked trials in which the variable being judged was manipulated while the other variable was held constant. Finally, participants were tested in a completely randomized design. Results in the blocked and randomized conditions were comparable.

As shown in Figure 2, judgments of mean relative phase varied linearly with actual mean relative phase. However,, as phase variability increased, 0° mean phase was increasingly confused with 30° mean phase. Furthermore, as illustrated in Figure 4, although mean judgments tracked actual relative phases very well, the variability of the judgments exhibited an inverted-U pattern. This meant that judgments of 90° relative phase, for instance, were far less reliable than judgments of 0° relative phase.

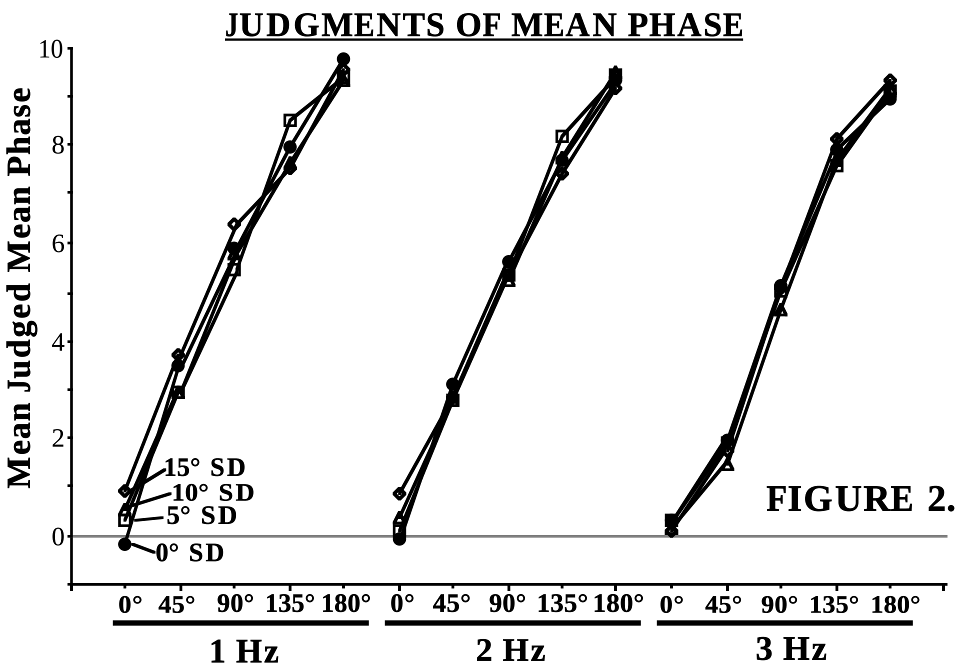
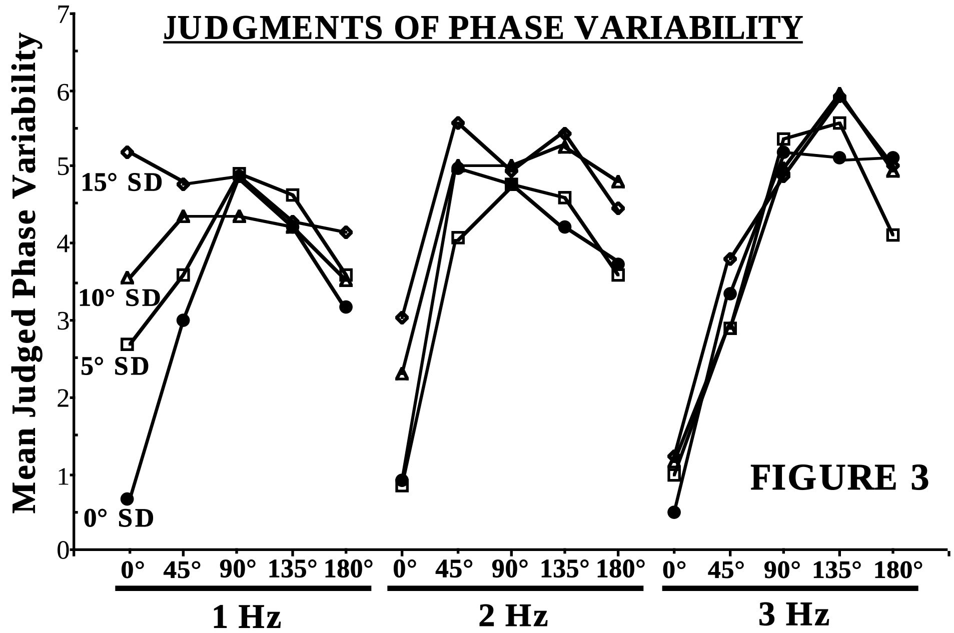


FIGURE 2.

We found that judgments of phase variability (or of the stability of movement) followed an asymmetric inverted-U function of mean relative phase, even with no phase variability in the movement as shown in Figures 3 and 4. This replicated the shape of the potential function in the HKB model. Movement at 0° relative phase was judged to be most stable. At 180°, movement was judged to be less stable. At intervening relative phases, movement was judged to be relatively unstable and maximally so at 90°. Levels of phase variability were not discriminated at relative phases other than 0° and 180° because those movements were already judged to be highly variable even with no phase variability. The standard deviations of judgments followed this same asymmetric inverted-U pattern as shown in Figure 4.



Finally, we investigated whether phase perception would vary in a way consistent with the finding in bimanual coordination studies of mode switching from 180° to 0° relative phase when the frequency was sufficiently increased. Also, movement studies revealed that increases in the frequency of movement yielded increases in phase variability at 180° relative phase but not at 0° relative phase. As shown in Figure 3, as frequency increased, movements at all mean relative phases other than 0° were judged to be more variable. This was true in particular at 180° relative phase. Furthermore, as shown in Figures 3 and 4, this occurred even when there was no phase variability in the movement. Also in this latter case (i.e 0 phase SD), frequency had no effect on judged levels of phase variability at 0° mean phase (although in cases of 5°, 10° and 15° phase SD, actual phase variability became less salient at higher frequencies). Again, mean phase was judged accurately on average, but as frequency increased, judgments tended to drop as shown in Figures 2 and 4. As also shown, the pattern of results for mean judgments of phase variability was replicated in the pattern of results for judgment variability both for judgments of mean phase and of phase variability.

Mean Results for 0 Phase SD Only (i.e. no actual phase variability)

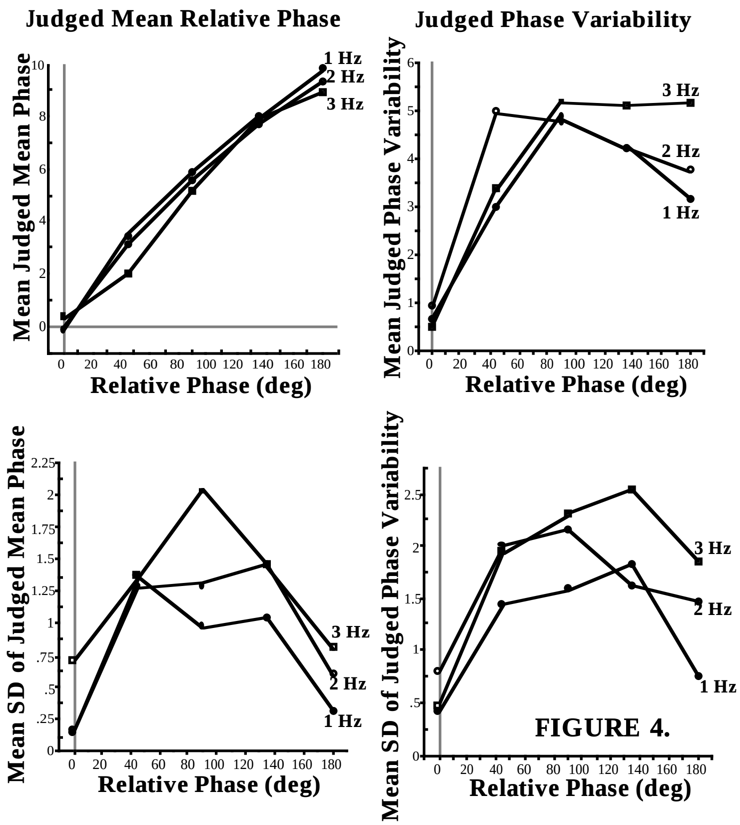


FIGURE 4.

These results were all consistent with the findings of the studies on bimanual coordination. **The asymmetric inverted-U pattern of the judgments in Figures 3 and 4 is essentially the same as the potential function of the HKB model.** The potential represents the relative stability of coordination or the relative effort of maintaining a given relative phase. The two functions match not only in the inverted-U shape centered around 90° relative phase, but also in the asymmetry between 0° and 180°. 180° is perceived to be less stable than 0° and increasingly so as frequency increases. This congruence of the movement and perception results supports the hypothesis that the relative stability of bimanual coordination is a function of phase perception and its stability.

Endpoints versus the trajectory

Many rhythmic tasks entail moving in synchrony with a discrete auditory pulse, that is, a metronome. Perhaps this has inspired a common intuition that the perception of rhythmic movements focuses on the endpoints of movement. Indeed, rhythmic movements could be generated using a mass-spring by simply switching the position of the EP discontinuously from one endpoint to the other (although Feldman and his co-authors explicitly reject this possibility (Feldman, Adamovich, Ostry & Flanagan, 1990)). If perception of relative phase depended only on relative positions at endpoints of motion, then stability only at 0° and 180° relative phase would be predicted. We investigated whether perception of relative phase focused on the endpoints of motion or instead, used the entire trajectory of oscillation. We could not use selective occlusion of portions of the trajectories to address this question because discontinuities at occlusion boundaries would perturb motion perception strongly. Instead, we chose to put phase variability selectively into portions of the oscillatory cycle as follows. See Figure 5.

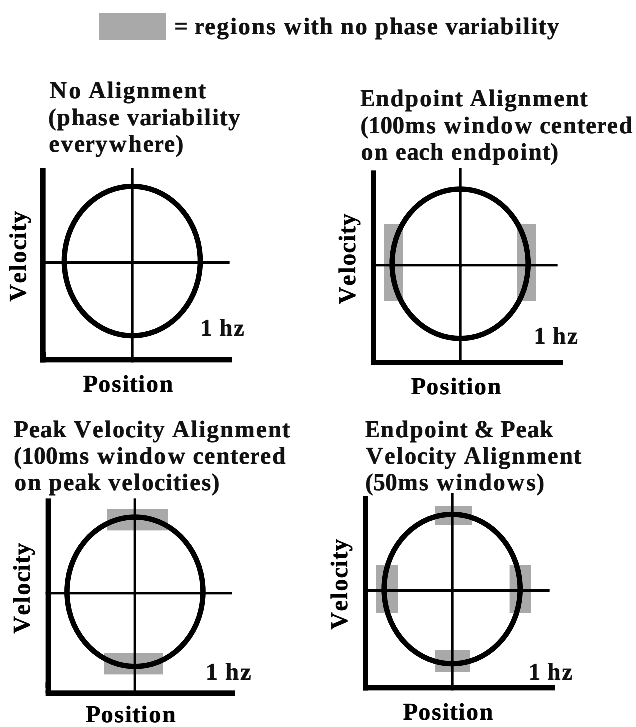
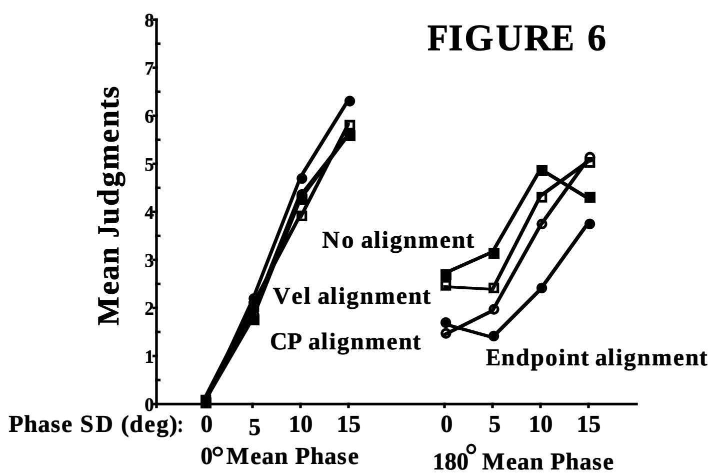


FIGURE 5.

First, we put relative phase variability throughout the cycles (No Alignment) as in previous studies. Second, we put it everywhere but in a 100ms window around each endpoint (Endpoint Alignment). Oscillation frequency was 1 Hz, so 20% of the cycle was free of phase variability. Third, we put it everywhere but in a 100ms window around each peak velocity (Velocity Alignment). Fourth, we put it everywhere but in a 50ms window around the endpoints and peak velocities (Critical Points Alignment). As before, 4 levels of phase variability were tested. (With a 20% reduction in the EA, VA, and CPA conditions, these were 0°, 4°, 8°, and 12° phase SD as opposed to 0°, 5°, 10°, and 15° phase SD in the NA condition). Only mean relative phases of 0° and 180° were tested. Ten observers judged phase variability as in the previous experiments.

If phase perception uses only the endpoints of oscillatory movement, then the increasing levels of phase variability should have been invisible in the Endpoint Alignment condition because there was no phase variability at the endpoints. The judgment curves should be flat. The logic was the same in the Velocity and Critical Points Alignment conditions under the assumption that perception focuses on peak velocities or on critical points.



As shown in Figure 6, the results did not confirm these predictions. Instead, they revealed that the entire trajectory is used to perceive relative phase. Looking at the 180° mean phase condition, removal of variability from endpoints decreased perceived variability somewhat (as it should have because it was 20% less), but the remaining portions of the cycle were still used to detect variations in phase variability. However, removal of variability from the peak velocities had no effect. Removal from both peak velocity and endpoints in the CPA condition had half the effect of removal from just endpoints, but in the CPA condition, the window around endpoints was half as large. That is, only 10% of the variability was removed around the endpoints, the other 10% was removed at the peak velocities where it had no effect. The conclusion was that the whole trajectory is used, but phase variability at peak velocity is invisible because movement there already looks variable. (The effect is similar to that for phase variability at 90° mean phase shown in Figure 3. Movement at 90° relative phase looks variable intrinsically, so actual phase variability cannot be resolved.) None of the alignment manipulations had any effect for movement at 0° mean phase. Accordingly, we concluded that the entire trajectory is used, but resolution of relative phase varies with relative velocity or the velocity difference between the two oscillators. (With 0° mean phase, relative velocity was always zero, or with noise, nearly so.) Phase perception becomes unstable as the relative velocity becomes large.

Next, we developed a model of bimanual coordination in which the role of phase perception is explicit. The goal was to account both for our phase judgment results and for results from previous motor studies.

Modeling the single oscillator

Our perception studies had been inspired originally by the HKB model. The HKB model is a first order dynamical model in which relative phase is the state variable. That is, the model describes relative phase behavior directly without reference to the behavior of the individual oscillators. However, the model was derived from a model formulated by Kay, Kelso, Saltzman and Schöner (1987) that did describe the oscillation of the limbs explicitly. In this latter model, the state variables are the positions and velocities of the two oscillators. To develop this model, Kay, et al. (1987) first modeled the rhythmic behavior of a single limb. In this and a subsequent study (Kay, Saltzman & Kelso, 1991), they showed that human rhythmic limb movements exhibit limit cycle stability, phase resetting, an inverse frequency-amplitude relation, a direct frequency-peak velocity relation, and, in response to perturbation, a rapid return to the limit cycle in a time that was independent of frequency. A dimensionality analysis showed that a second-order dynamic with small amplitude noise is an appropriate model. The presence of a limit cycle meant the model should be nonlinear and a capability for phase resetting entailed an autonomous dynamic. Kay, et al. (1987) captured these properties in a 'hybrid' model that consisted of a linear damped mass-spring with two nonlinear damping (or escapment) terms, one taken from the van der Pol oscillator and the other taken from the Rayleigh oscillator (hence the 'hybrid') yielding:

$$\ddot{x} + b\dot{x} + \alpha\dot{x}^3 + \gamma x^2\dot{x} + kx = 0 \qquad (1)$$

This model was important because it captured the principle dynamical properties exhibited by human rhythmical movements. However, the relation between the terms of the model and known components of the human movement system was unclear. The damped mass-spring was suggestive of Feldman's λ-model which represents a functional combination of known muscle properties and reflexes. Nevertheless, in the hybrid model, the functional realization of the nonlinear damping terms was unknown.

Following a strategy described by Bingham (1988), Bingham (1995) developed an alternative model to the hybrid model. All of the components of the new model explicitly represented functional components of the perception/action system. First, the model incorporated the λ-model, that is, a linear damped mass-spring:

$$\ddot{x} + b\dot{x} + k(x - x_{ep}) = 0$$

This mass-spring must be driven to generate rhythmic movement. This can be achieved by moving the EP:

$$\ddot{x} + b\dot{x} + k(x) = k(x_{ep}(t))$$

If the timing is imposed, then the result is the standard forced oscillator:

$$\ddot{x} + b\dot{x} + k(x) = c(\sin(t)), \quad c = f(k)$$

The problem is that this is a nonautonomous dynamic, that is, a dynamic that would not exhibit phase resetting. Another problem is that limb movements are known to exhibit organizations that are both energetically optimal and stable (e.g. Diedrich & Warren, 1995; Margaria, 1976; McMahon, 1984). Both energy optimality and stability are achieved by driving a damped mass-spring at resonance, that is, with the driver leading the oscillator by 90°. Accordingly, Hatsopoulos and Warren (1996) suggested that this strategy might be used in driving the Feldman mass-spring. Bingham (1995) solved these problems by replacing time in the driver by the perceived phase of the oscillator. That is, instead of **c sin(t)**, the driver is **c sin(ϕ)**, where ϕ is the phase. Because ϕ (**= f[x, dx/dt]**) is a (nonlinear) function of the state variables, that is, the position and velocity of the oscillator, the resulting dynamic is autonomous. The perceptually driven model is:

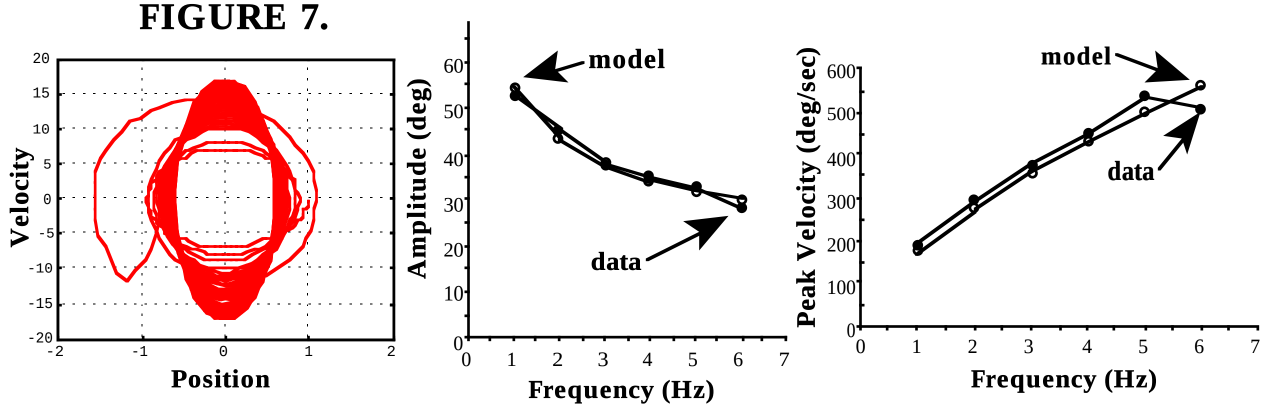
$$\ddot{\mathbf{x}} + \mathbf{b}\,\dot{\mathbf{x}} + \mathbf{k}\,\mathbf{x} = \mathbf{c}\,\mathbf{sin}[\phi] \quad (2)$$

where

$$\phi = \mathbf{arctan}\left[\frac{\dot{\mathbf{x}}_{\mathbf{n}}}{\mathbf{x}}\right], \ \dot{\mathbf{x}}_{\mathbf{n}} = \dot{\mathbf{x}}/\sqrt{\mathbf{k}} \ \text{ and } \mathbf{c} = \mathbf{c}\,(\mathbf{k}).$$

The amplitude of the driver is a function of the stiffness. Bingham (1995) showed that this oscillator yields a limit cycle. This is also shown in Figure 7 by rapid return to the limit cycle after a brief perturbing pulse. As also shown, the model exhibits the inverse frequency-amplitude and direct frequency-peak velocity relations as frequency was increased from 1 hz to 6 hz. These relations are apparent in the first panel of Figure 7 showing a phase plot generated by gradually increasing the frequency of the oscillator. In the second and third panels, the model is compared to the human movement data reported by Kay, et al. (1987).

FIGURE 7.



Finally, the model exhibits a pattern of phase resetting that is similar to that exhibited by the hybrid oscillator as shown in Figure 8. The model is phase delayed by a delaying perturbation and phase advanced by an advancing perturbation. However, as shown in the second panel by the graph taken from Kay, et al. (1991), human participants exhibited phase advance in response to all perturbations. Following a suggestion in Kay, et al., we modified the phase driven oscillator model to include a momentary increase in stiffness in response to a perceived departure from the limit cycle as a result of perturbation (see the first panel of Figure 7). $\mathbf{k}$ in equation (2) above was changed to

$$\mathbf{k} = \mathbf{k_i} + \gamma \left| \mathbf{e_t} - \mathbf{e_i} \right| \quad \text{where } \mathbf{e_n} = (\mathbf{v_n^2} + \mathbf{x^2})^{.5}$$

is both the radius of the trajectory on the phase plane and a measure of the energy of motion. Thus, stiffness was incremented in proportion to the change in the radial coordinate in phase space. (The other coordinate is the phase angle, ϕ.) Note that equation (2) remains autonomous because $\mathbf{e_n}$ is a function of state variables $\mathbf{x}$ and $\mathbf{v}$. $\mathbf{e_n}$ is hypothesized as another perceptual variable in addition to ϕ. The result, shown in the third panel of Figure 8, was a pattern of phase response similiar to the human data.

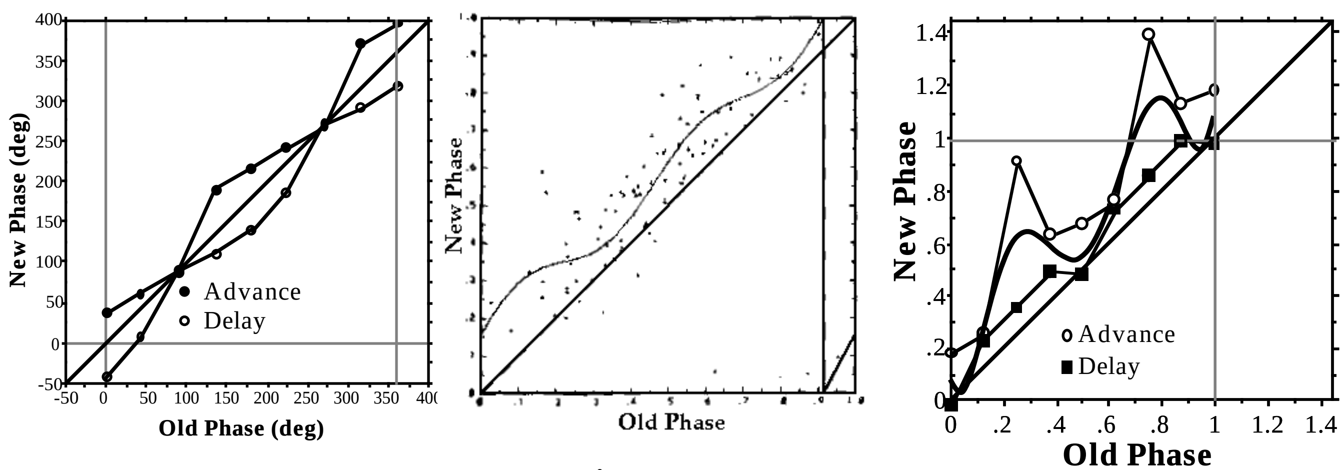


Figure 8.

Goldfield, Kay and Warren (1993) found that human infants were able to drive a damped mass-spring at resonance. The system consisted of the infant itself suspended from the spring of a "jolly bouncer" which the infant drove by kicking. This essentially instantiates the phase driven oscillator model and shows that even infants can use perceived phase to drive such an oscillator at resonance. We hypothesize that all adult rhythmic limb movements are organized in this way.

Modeling coupled oscillators

With this model of a single oscillating limb, we were ready to model the coupled system. Kay, et al. (1987) had modeled the coupled system by combining two hybrid oscillators via a nonlinear coupling:

$$\begin{aligned}
&\ddot{\mathbf{x}}_1 + \mathbf{b}\,\dot{\mathbf{x}}_1 + \alpha\,\dot{\mathbf{x}}_1^3 + \gamma\,\mathbf{x}_1^2\ \dot{\mathbf{x}}_1 + \mathbf{k}\,\mathbf{x}_1 = \\
&\qquad (\dot{\mathbf{x}}_1 - \dot{\mathbf{x}}_2)[\,\mathbf{a} + \mathbf{b}\,(\mathbf{x}_1 - \mathbf{x}_2\,)^2] \\
&\ddot{\mathbf{x}}_2 + \mathbf{b}\,\dot{\mathbf{x}}_2 + \alpha\,\dot{\mathbf{x}}_2^3 + \gamma\,\mathbf{x}_2^2\,\dot{\mathbf{x}}_2 + \mathbf{k}\,\mathbf{x}_2 = \\
&\qquad (\dot{\mathbf{x}}_2 - \dot{\mathbf{x}}_1)[\,\mathbf{a} + \mathbf{b}\,(\mathbf{x}_2 - \mathbf{x}_1\,)^2] \qquad (\mathbf{3})
\end{aligned}$$

This model required that people simultaneously perceive the instantaneous velocity difference between the oscillators as well as the instantaneous position differences so that both could be used in the coupling function. This model did yield the two stable modes (namely, 0° and 180° relative phase) at frequencies near 1 hz, and mode switching from 180° to 0° relative phase at frequencies between 3 hz and 4 hz.

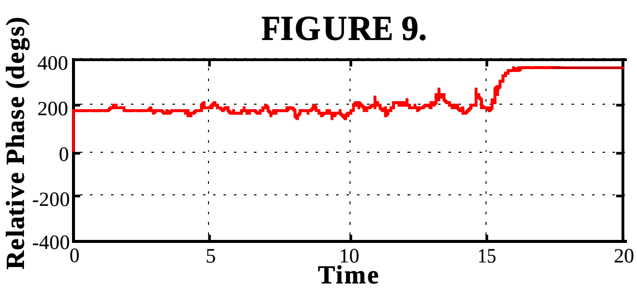
We have proposed an alternative model (Bingham, 2001; Bingham & Collins, in preparation) in which two phase driven oscillators are coupled by driving each oscillator using the perceived phase of the other oscillator multiplied by a term, P, that represents the perceived relative phase. P is computed as the sign of the product of the two drivers. P simply indicates at each instant whether the two oscillators are moving in the same direction (**sgn = +1**) or in opposite directions (**sgn = –1**). The model is:

$$\begin{aligned}
&\ddot{\mathbf{x}}_1 + \mathbf{b}\,\dot{\mathbf{x}}_1 + \mathbf{k}\,\mathbf{x}_1 = \mathbf{c}\ \mathbf{sin}(\phi_2)\ P_{\mathbf{ij}} \\
&\ddot{\mathbf{x}}_2 + \mathbf{b}\,\dot{\mathbf{x}}_2 + \mathbf{k}\,\mathbf{x}_2 = \mathbf{c}\ \mathbf{sin}(\phi_1)\ P_{\mathbf{ji}} \qquad (\mathbf{4})
\end{aligned}$$

where

$$P = \mathbf{sgn}(\mathbf{sin}(\phi_1)\ \mathbf{sin}(\phi_2) + \alpha(\dot{\mathbf{x}}_{\mathbf{i}} - \dot{\mathbf{x}}_{\mathbf{j}})\ N_{\mathbf{t}}) \qquad (5)$$

As shown in equation (5), the product of the two drivers is incremented by a Gaussian noise term with a time constant of 50 ms and a variance that is proportional to the velocity difference between the oscillators. This noise term reflects known sensitivities to the directions of optical velocities (De Bruyn & Orban, 1988; Snowden & Braddick, 1991) and is motivated by the results from the phase perception experiment described above. We found that relative phase appeared more variable as the relative velocity of the two oscillators increased. It is important to note that this noise term does not imply that the velocity difference is perceived, but only that the ability to resolve the relative direction of movement is affected by the relative speeds. This model also yields only two stable modes (at 0° and 180° relative phase) at frequencies near 1 hz, and yields mode switching from 180° to 0° relative phase at frequencies between 3 hz and 4 hz. This is shown in Figure 9 where the oscillators were started at 180° at 1 hz and then, as the frequency was increased, exhibited increasing variability in relative phase eventually switching to 0° (360°) at a frequency of about 4 hz. After the switch, the variability decreased strongly.



Furthermore, the model predicts our results for judgments of mean relative phase and of phase variability. Judged mean phase is produced by integrating P over a moving window of width σ (= 2 s) to yield $\mathbf{P_{JM}}$:

$$\mathbf{P_{JM}} = \frac{\int_{\mathbf{t}-\sigma}^{\mathbf{t}} \mathrm{P}\, \mathbf{dt}}{\sigma} \quad (\mathbf{6})$$

Judged phase variability is predicted by integrating $(\mathrm{P}-\mathbf{P_{JM}})^{\mathbf{2}}$ over the same window to yield $\mathbf{P_{JV}}$:

$$\mathbf{P_{JV}} = \frac{\int_{\mathbf{t-\sigma}}^{\mathbf{t}} [\mathrm{P} - \mathbf{P_{JM}}]^2 \mathbf{dt}}{\sigma} \quad (7)$$

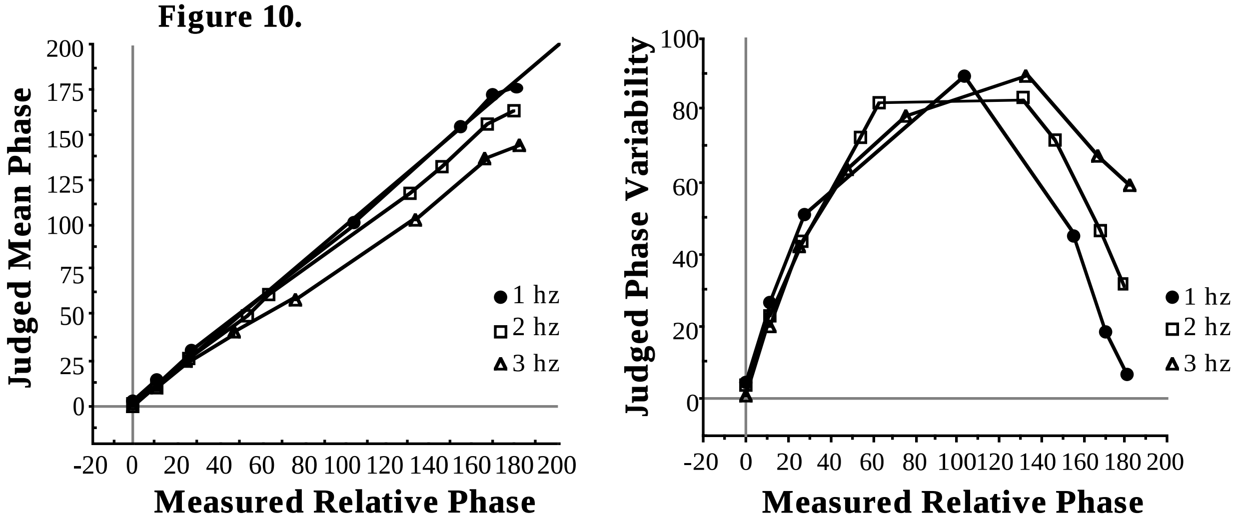
$\mathbf{P_{JM}}$ varies linearly with actual mean phase and $\mathbf{P_{JV}}$ yields an asymmetric inverted-U as a function of actual mean phase as shown in Figure 10. As also shown, $\mathbf{P_{JM}}$ and $\mathbf{P_{JV}}$ behave as did the respective judgments in response to increases in the frequency of oscillation. Compare Figure 10 to Figure 4. The model reproduced the results from the judgments studies where observers judged either mean relative phase or phase variability. (These results were obtained using an external forcing to drive the system to phases other than 0° and 180° (Tuller & Kelso, 1989):

$$\ddot{\mathbf{x}}_{\mathbf{1}} + \mathbf{b}\,\dot{\mathbf{x}}_{\mathbf{1}} + \mathbf{k}\,\mathbf{x}_{\mathbf{1}} = \mathbf{c}\ \mathbf{sin}\ (\phi_2)\ \mathrm{P}_{\mathbf{ij}} + \mathbf{d}\ \mathbf{sin}(\sqrt{\mathbf{k}}\ \mathbf{t})$$

$$\ddot{\mathbf{x}}_{\mathbf{2}} + \mathbf{b}\,\dot{\mathbf{x}}_{\mathbf{2}} + \mathbf{k}\,\mathbf{x}_{\mathbf{2}} = \mathbf{c}\ \mathbf{sin}\ (\phi_1)\ \mathrm{P}_{\mathbf{ji}} + \mathbf{d}\ \mathbf{sin}(\sqrt{\mathbf{k}}\ \mathbf{t} + \phi_{\mathbf{R}})$$

where $\phi_{\mathbf{R}}$ was manipulated to achieve particular relative phase relations.)

Figure 10.



There are two aspects of the perceptual portions of the model that should be emphasized. First, there are actually two perceptible properties entailed in the model. The two are very closely related, but they are distinct. The first is the phase of a single oscillator. The perception thereof is entailed in the single oscillator model. This is, of course, incorporated into the coupled oscillator model. The second perceptible property is relative phase. This latter property brings us to the second aspect of the model to be noted. This is especially important.

This model is being used to model performance in two different tasks, one is a coordinated movement task and the other is a judgment task. Equation (5) represents the way the perception of relative phase plays a role in the coordinated movement task. This is in terms of the momentary value of P, that is, whether the oscillators are perceived to be moving in the same or in opposite directions at a given moment in time. This modifies the driving effect of the respective perceived phases. In contrast, equations (6) and (7) represent the way the perception of relative phase plays a role in the judgment tasks. In this case, the behavior of P is assessed (that is, integrated) over some window of time that is large enough to span one or two cycles of movement. So, the two tasks are connected by a single perceptible property, but the way the property is evaluated and used is task-specific.

The model is representative of nonlinear dynamics: complex behavior emergent from simple dynamic organization. The model captures both the movement results and results of perceptual judgments. Two relatively simple equations (4) capture the fundamental properties of human rhythmic movements: limit cycle stability, phase resetting, inverse frequency-amplitude and direct frequency-peak velocity relationships, the stable modes and mode transitions and the increasing patterns of instability leading up to mode transition. With the addition of two more simple equations (6) and (7) computing a mean and a variance, the model accounts for the results for perceptual judgments of mean relative phase and of phase variability and the ways these vary with the frequency of movement. All this from a model with 6 parameters (**k, b, c,** α**,** γ**,** and σ), four of which are fixed and one, **k,** is varied to generate variations in frequency of movement. (Note: because **c=f(k), c** varies with k but once the scaling of **c** is fixed, this does not represent an extra degree of freedom.)

This model builds on the previous results of Kay et al. (1987) and Kay et al. (1991) which revealed fundamental dynamic properties of human movement. Those properties are captured by the new model as they were by previous models. However, unlike the previous models, the new model is an explicit perception / action model. Its components are interpretable in terms of known components of the perception / action system. It explicitly represents the perceptual coupling that is well recognized to be fundamental to the coordination task and the resulting bimanual behaviors.

ϕ and τ

We began by studying the visual perception of relative phase and indeed, we found that the pattern of judgments was consistent with the pattern of results from studies on rhythmic limb movement. We should mention that we have also replicated the visual phase perception results in a haptic phase perception task (Wilson, Craig & Bingham, in preparation). However, when we turned to modeling, it was necessary to consider a related but different variable, namely, phase ϕ. Both phase and relative phase are required for the model of bimanual coordination. Phase is required in the context of each single oscillator while relative phase is required in addition for their coordination. (In fact, for the single oscillator, we ultimately required a third perceptible variable, the 'energy' **e**.)

It is the phase that is most relevant in the current context. As a perceptible variable, phase ϕ is very similar to τ. However, they are not the same. Both are derived as a ratio of position and velocity , that is, variables that may be state variables in a dynamic system. In terms of these variables,

$$\phi = \mathbf{arctan}\left[\frac{\mathbf{1}}{\tau \cdot \mathbf{f}}\right] \quad (\mathbf{8})$$

but this is not entirely appropriate because there are constraints on the underlying space for τ that do not apply for ϕ and visa versa. τ is defined to describe motion relative to (usually an approach to) an origin along the positive half of the x axis, that is, the range of x includes positive values and zero. Alternatively, ϕ is defined relative to the equilibrium point of an oscillator and x takes on both positive and negative values around an origin located at the equilibrium point. Then, while both are timing or time relative variables, only τ is a temporal variable. Dimensionally, ϕ is dimensionless, more specifically, angular. As shown in equation (8), the ratio of state variables is normalized by frequency, **f**. Thus, unlike τ, ϕ is defined relative to a cyclic or periodic structure. The oscillatory dynamic is intrinsic to ϕ, but not to τ.

So, ϕ is certainly not the same as τ, but they reflect similar strategies for understanding and modeling perception / action systems. Both are derived in terms of ratios of spatial variables so both avoid the classic measurement problem in space perception. Relatedly, both are intrinsic timing variables and so relate directly to timing of behavior. Finally, both are derived from variables that can play the role of state variables in a dynamic system. This means that they can be used as drivers to build autonomous dynamical organizations to model stable, self-organizing perception / action systems.

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